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Volume II

PROPULSION NOZZLE STUDIES

Volume II
DESIGN OF MAXIMUM THRUST NOZZLE-BASE-BOATTAIL CONTOURS

THERMAL SCIENCES AND PROPULSION CENTER
PURDUE UNIVERSITY
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A production-type computer program ~~has been~~ was developed for the design of maximum thrust axisymmetric nozzle-base-boattail contours. The flow field analysis is based on the governing equations for the rotational flow of a frozen or equilibrium gas mixture. The contour optimization is based on the direct optimization of a nonlinear function of several (three or less) geometric variables.
The nozzle and boattail contours are modeled as a second-order polynomial

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and a cone, respectively. For fixed initial expansion contours and nozzle and boattail lengths, the second-order polynomial nozzle contour is uniquely specified by the nozzle throat attachment angle and the nozzle exit lip radius, and the conical boattail is uniquely specified by the boattail exit lip radius. These three independent parameters are varied to determine the unique nozzle-base-boattail configuration that yields maximum thrust.

Three methods are included in the program to determine the maximum thrust contour. Each method requires an initial estimate of the geometry that produces maximum thrust. The methods are derivative methods that perturb the wall geometry parameters to determine approximate local derivatives. This information is used to calculate a search direction and a step length in the optimization variable space. Successive applications of the procedure modify the contour parameters until the maximum thrust configuration is obtained.

The first method is axial iteration. It treats the thrust as a function of up to three independent coordinates, and moves in the direction of each coordinate individually, using first and second partial derivative approximations to determine the step length. The second method is a gradient technique, the method of steepest descent. It computes an approximation to the gradient of the thrust function and steps an arbitrary step length in the direction of the gradient. The third technique is Newton's method, a second-derivative method. Newton's method computes first and second-derivatives of the thrust function, defining both the direction and the step length for the optimization step.

The three methods were first applied to the design of maximum thrust nozzle contours. Each of the methods converges efficiently to the maximum thrust nozzle contour. Newton's method and axial iteration converge very quickly, in approximately twenty nozzle flow field calculations or less. The method of steepest descent requires as much as twice as many flow field calculations as the other methods, but proceeds to the exact maximum by converging to a point whose gradient is zero. The nozzle contours from these methods are shown to yield nearly identical thrusts to those predicted by maximization using the calculus of variations, affirming the validity of the direct optimization approach.

The three methods were also applied to the design of nozzle-base-boattail configurations. The three-dimensional optimizations were found to be practical for finding the maximum thrust attainable for a nozzle-base-boattail assembly. Each of the three methods converged efficiently to the maximum thrust nozzle-base-boattail contour.

PREFACE

This final report was submitted by the Thermal Sciences and Propulsion Center of Purdue University, under Contract No. F33615-73-C-2010. The effort was sponsored by the Air Force Aero Propulsion Laboratory, Air Force Wright Aeronautical Laboratories, Air Force Systems Command, Wright-Patterson AFB, Ohio under Project No. 3012, Task 301213, and Work Unit 30121306 with P. J. Hutchison/AFAPL/RJT as Project Engineer. Jeffrey G. Allman and Joe D. Hoffman of Purdue University were technically responsible for the work.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	Speed of sound.
I	Coordinate of right-running Mach line.
J	Coordinate of left-running Mach line.
M	Mach number.
n	Number of independent optimization variables.
p	Static pressure.
P	Stagnation pressure.
R	Gas constant.
t	Static temperature.
T	Stagnation temperature.
u	x-component of velocity.
v	y-component of velocity.
x	Axial coordinate.
y	Radial coordinate.
α	Mach angle.
γ	Specific heat ratio.
δ	Equals 0 for planar flow and 1 for axisymmetric flow.
η	Correlation factor for base pressure model.
θ	Flow angle.
ρ	Density or radius of curvature.

Subscripts

ab	Boattail attachment point.
amb	Ambient property.
an	Nozzle throat attachment point.
b	Base region property.
eb	Boattail exit lip point property.
en	Nozzle exit lip point property.
t	Throat property.
td	Downstream of the throat.
tu	Upstream of the throat.
x	Partial derivative with respect to x.
y	Partial derivative with respect to y.
$\pm$	Left- or right-running Mach line, respectively.

Superscripts

*	Critical property, or stationary point.

SECTION I

INTRODUCTION

Current design techniques for propulsive nozzle contours assume that the nozzle exhausts into a static region having a specified ambient pressure. The design objective is to select that contour which yields maximum thrust for that ambient pressure. As illustrated in Fig. 1, the thrust producing region is much more complex. The nozzle is generally installed in an airframe with an annular base region between the nozzle lip and the airframe lip. Often the aft portion of the airframe, referred to as the boattail, is contoured to lessen the thrust loss associated with the low pressure base region. Consideration of boattail and base thrusts modifies the optimum nozzle contour from that for an isolated nozzle.

A technique for designing conical nozzles in a cylindrical boattail including the effects of the base region was developed by Byington and Hoffman (1), making possible performance increases of up to two percent. No analogous technique is available for designing contoured nozzles including the effects of the base region or for designing contoured nozzles and contoured boattails including the effects of the base region.

The objective of the present investigation is to develop an efficient method for the design of maximum thrust contours for nozzle-base-boattail assemblies, such as illustrated in Fig. 1.

The nozzle and boattail contours must satisfy certain geometric and physical constraints; for example, fixed overall length for both contours, specified throat radius and initial expansion contour for the nozzle, specified outer radius and initial expansion contour for the boattail, and specified initial conditions for the nozzle and boattail flow fields. Substantial effort has been directed to procedures for maximum thrust nozzle design, and several computer programs for nozzle design based on the calculus of variations have been developed. The difficulty with the calculus of variations approach is that if the gas dynamic model is modified or the general geometry of the thrust producing region is changed, the entire optimization analysis and computer program must be reworked, a task requiring several man years of effort.

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(1) C.M. Byington, Jr. and J.D. Hoffman, "Effects of Base Pressure on Conical Thrust Nozzle Optimization," AIAA Journal, Vol. 7, No. 3, March 1970, pp. 380-382.

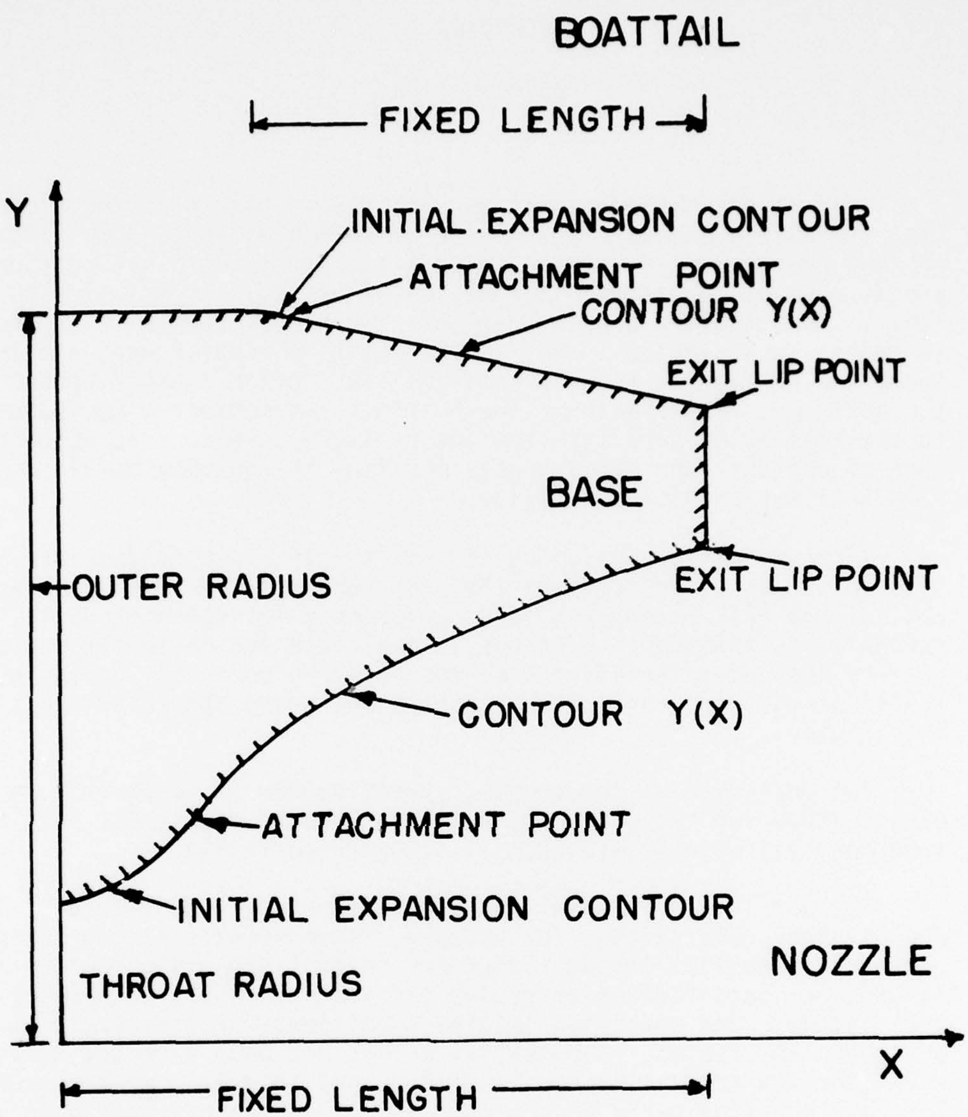

Figure 1. Combined Nozzle-Base-Boattail Optimization Model

The approach taken in this investigation is to develop an optimization procedure that is valid for any gas dynamic model and a variety of geometric configurations. To achieve that goal, the nozzle wall contour was assumed to be a second-order polynomial of the form $y = a + bx + cx^2$, and the boattail contour was assumed to be a first-order polynomial of the form $y = a + bx$. Nonlinear programming methods (2) are employed to vary the polynomial coefficients a, b, and c, seeking the nozzle and boattail contours that yield maximum thrust.

Two questions must be answered. First, do wall contours specified by polynomials yield results comparable to those obtained by the calculus of variations? Second, which nonlinear programming methods most efficiently solve the nonlinear programming problem?

The present analysis is limited to applications where the external flow over the boattail is supersonic. Viscous effects are neglected in both the nozzle flow field and the boattail flow field. Consequently, transonic flow effects and flow separation effects on the boattail are both neglected. The resulting computer program is thus limited to supersonic flight speeds at design conditions where separation does not occur. The optimization procedure, however, does not depend on the flow field model, and it should be possible to apply the procedure to any flight speed, including viscous effects, when rapid and accurate numerical methods for determining the corresponding flow field become available.

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[2] D.M. Himmelblau, Applied Nonlinear Programming, McGraw-Hill Book Co., New York, 1972.

SECTION II

ANALYSIS

The design of maximum thrust contours for any thrust producing element requires the ability to analyze the flow field in that particular element, and an optimization method for determining the maximum thrust contour. The flow field analysis procedure and the optimization methods employed in this investigation are discussed in this section.

1. Flow Field Analysis

The nozzle-base-boattail assembly geometry is illustrated in Fig. 1. The initial expansion contours for both the nozzle and the boattail are assumed to be circular arcs. The nozzle contour is assumed to be a second-order polynomial of the form

$$y(x) = a + bx + cx^2 \tag{1}$$

and the boattail contour is assumed to be a first-order polynomial of the form

$$y(x) = a + bx \tag{2}$$

The above choices for the nozzle and boattail contours require the determination of the coefficients a, b, and c for the nozzle contour and the coefficients a and b for the boattail contour. These coefficients are uniquely determined by specifying the nozzle throat attachment angle θ_{an}, the nozzle exit radius y_{en}, the boattail exit radius y_{eb}, and by requiring that the polynomial contours attach smoothly to the circular arc initial-expansion contours (see Fig. 1). These three parameters uniquely specify a nozzle-base-boattail configuration. The optimization procedure searches through the allowable ranges of these three parameters to determine the particular set of values that yields the maximum total thrust on the nozzle-base-boattail assembly.

The gas dynamic model is that for the steady axisymmetric flow of an inviscid fluid in the absence of work, heat transfer, and body forces. Such a flow is isentropic. The governing partial differential equations are the following (3).

$$\rho u_x + \rho v_y + u\rho_x + v\rho_y + \delta\rho v/y = 0 \tag{3}$$

$$\rho u u_x + \rho v u_y + p_x = 0 \tag{4}$$

$$\rho u v_x + \rho v v_y + p_y = 0 \tag{5}$$

$$u p_x + v p_y - a^2 u \rho_x - a^2 v \rho_y = 0 \tag{6}$$

Equation (3) is the continuity equation, equations (4) and (5) are the x and y Euler momentum equations, and equation (6) is a form of the speed of sound equation for an isentropic flow.

This system of four partial differential equations may be replaced by a system of characteristic and compatibility equations, which are ordinary differential equations (see Appendix A). For isentropic flow, the acoustic speed is a function of the local static pressure and density. The composition is assumed to be a general mixture, either fixed (frozen) in composition, or in chemical equilibrium. The gas dynamic model requires that the entropy, the stagnation pressure, and the stagnation enthalpy remain constant along streamlines, although these properties may vary between streamlines. A conventional predictor-corrector second-order numerical integration procedure (4) is employed to integrate the characteristic and compatibility equations, starting from a supersonic initial-value line.

Several procedures have been developed for analyzing the transonic flow in the throat region of convergent-divergent nozzles. Kliegel's method (5) is employed in the present study because it is accurate and easy to use. Kliegel's method stems from a study by Hall (6). Both solutions involve expansions for the velocity components u and v in inverse powers of R, the ratio of the upstream throat wall radius of curvature ρ_{tu} to the throat radius y_t ($R = \rho_{tu}/y_t$). Hall used a straight expansion in inverse powers of R. However, the resulting series is divergent for $R < 1$. Kliegel modified Hall's approach, developing an expansion in inverse powers of $(R + 1)$, which is convergent for $R < 1$. The complete third-order axisymmetric solution for the velocity components derived by Hall and Kliegel is presented in Appendix B. Thus, the velocity components along an initial-value line

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(3) M.J. Zucrow and J.D. Hoffman, Gas Dynamics, Vol. I, John Wiley and Sons, New York, 1976, pp. 549.

(4) M.J. Zucrow and J.D. Hoffman, Gas Dynamics, Vol. I, John Wiley and Sons, New York, 1976, Chapter 12.

(5) J.R. Kliegel and J.N. Levine, "Transonic Flow in Small Radius of Curvature Nozzles," AIAA Journal, Vol. 7, No. 7, July 1969, pp. 1375-1378.

(6) I.M.Hall, "Transonic Flow in Two-Dimensional and Axially-Symmetric Nozzles," Quarterly Journal of Mechanics and Applied Mathematics, Vol. XV, Pt. 4, 1962, pp. 487-508.

may be calculated, defining a starting line for the numerical solution of the supersonic portion of the nozzle flow field by the method of characteristics.

The nozzle flow field is illustrated in Fig. 2. Contour AA' is the circular arc initial expansion contour. Point D is the nozzle throat attachment point where the maximum thrust contour DF attaches smoothly to the initial expansion contour. The maximum thrust nozzle contour is obtained by varying both the location of point D along the initial expansion contour and the contour between points D and F. The initial-value line AB is obtained by applying Kliegel's method, or by defining the line point by point from any transonic flow analysis. Starting from the initial-value line, the method of characteristics is applied to calculate the flow field in region ABCFA. Region ABCEDA is termed the kernel, and is affected solely by the initial-value line properties and the nozzle initial-expansion contour. Region DEFD, which is influenced by the nozzle wall contour, is the flow field region bounded by the right-running Mach line DE originating on the wall contour at the attachment point D and the left-running Mach line EF passing through the nozzle exit lip point F.

The analysis program employed in the study is a modification of the program developed by Pasley and Hoffman (7). That program was modified to minimize the computational effort during optimization, where many similar flow fields are computed. This is particularly pertinent in the nozzle, where much computational effort is spent calculating the flow field in the kernel. In general, the nozzle throat attachment angle θ_{an} corresponding to point D has some minimum value below which the optimization search will never proceed. The portion of the kernel upstream of that point is fixed and need not be recalculated as point D and the contour DF are varied during the optimization search. That minimum nozzle throat attachment point is indicated as point G on Fig. 2. Consequently, right-running Mach line GH may be stored and employed as the initial-value line for subsequent nozzle flow field calculations. The secondary-start line, as this new initial-value line is called, may save as much as 50 percent of the computational effort required for a nozzle flow field analysis. Care must be exercised, however, not to compute the secondary start line at too large an angle, since subsequent nozzle flow field calculations must have throat attachment angles greater than that of the secondary-start line. The program is not capable of restarting at any nozzle throat attachment angle less than or equal to that of the secondary-start line.

The left-running Mach line IJ is the initial-value line for the boattail flow field analysis. All of the flow properties along the initial-value line must be known from a prior analysis of the flow

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(7) S.A. Pasley and J.D. Hoffman, "Flow Field Analysis of a Nozzle-Boattail System," Air Force Aero Propulsion Laboratory, AFAPL-TR-74-38, 1974.

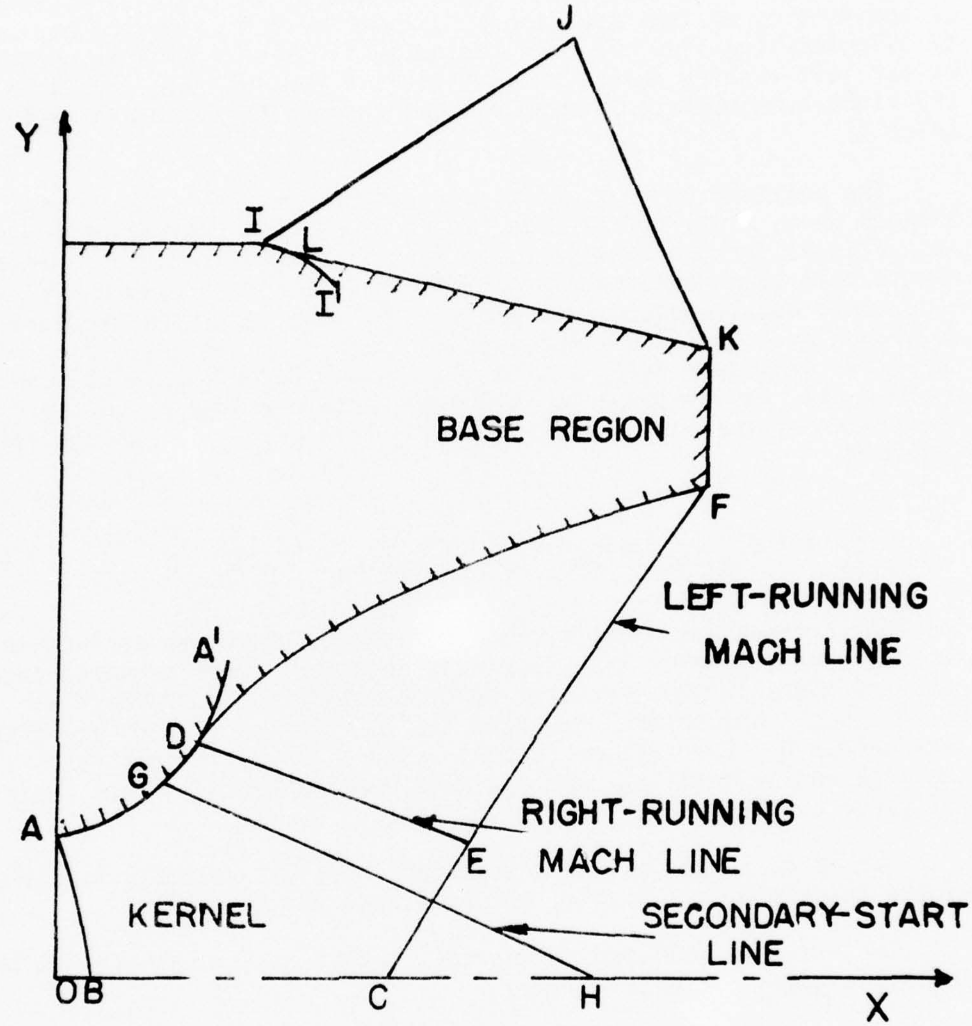

Figure 2. Combined Nozzle-Base-Boattail Flow Field Model

around the forward section of the vehicle. If such information is unavailable, a uniform flow initial-value line may be employed. Contour II' is the circular arc initial expansion contour, and point L is the point where the conical boattail contour KL attaches to the initial expansion contour II'. The external flow field is assumed to be supersonic, so that the method of characteristics may be employed to calculate the flow field in region IJKI. That region is bounded by the left-running Mach line IJ, which is the initial-value line, and the right-running Mach line JK passing through the boattail exit lip point K.

The boattail contour and nozzle contour may have a base region between them, as illustrated in Fig. 2. The total thrust on the vehicle must account for the base pressure in this annular region. Several models have been developed for predicting the base pressure aft of a supersonic nozzle-boattail assembly. The empirical base pressure model developed by Addy (8) was employed in the present investigation. A curve fit of Addy's data, similar to that employed by Byington and Hoffman (1), was employed to relate the base pressure p_b to the exit conditions of the nozzle and boattail. This curve fit has the following form.

$$\frac{p_b}{p_{eb}} = f(p_{en}/p_{eb})\eta(M_{eb})\eta(M_{en})\eta(T_{eb}/T_{en})\eta(\theta_{en}) \tag{7}$$

The four η functions are correlation factors, which are applied to the nominal case for which $f(p_{en}/p_{eb})$ was obtained. Each factor accounts for variations in the parameter denoted within the corresponding parentheses. The nominal case and the correlation factors are discussed in Appendix C. Correlation factors based on specific heat ratio γ and the base region radii y_{en} and y_{eb} have been neglected, since their influence on the base pressure is slight.

The total thrust of the nozzle-base-boattail assembly is the sum of the thrusts of the nozzle, the base, and the boattail.

The program developed during this investigation can perform the following types of calculations.

1. Analysis of a specified nozzle-base-boattail assembly.
2. Design of a maximum thrust nozzle contour.
3. Design of a maximum thrust boattail contour.
4. Design of a maximum thrust nozzle-base-boattail contour.

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(8) A.L. Addy, "Analysis of the Axisymmetric Base-Pressure and Base-Temperature Problem with Supersonic Interacting Freestream-Nozzle Flows Based on the Flow Model of Korst, et al., Part I: A Computer Program and Representative Results for Cylindrical Afterbodies," Advanced Systems Laboratory, Redstone Arsenal, Alabama, Report No. RD-TR-69-12, July 1969.

2. Optimization Procedures

The nozzle and boattail flow fields are entirely independent, and are calculated separately. Thus, the nozzle and boattail contour parameters are related only when considering a nozzle-base-boattail assembly, because they influence base pressure and hence total thrust on the assembly.

The nozzle typically provides the greatest thrust contribution to the vehicle. The boattail and base regions are normally contoured in such a way as to minimize the low pressure base region and boattail divergence loss effects.

The approach taken in this investigation is to model the wall contour geometry as a second-order polynomial for the nozzle contour and a first-order polynomial for the boattail contour (three independent parameters). These parameters are systematically varied to find the maximum thrust nozzle-base-boattail contour.

The nozzle-base-boattail assembly is first modeled as a two independent variable problem by fixing the width of the base region to be the minimum allowed width acceptable in the final design (which may be zero if a sharp trailing edge is permissible). Consequently, specification of the nozzle exit lip radius y_{en} fixes the value of the boattail exit radius y_{eb}, so that the boattail contour is completely specified. The nozzle throat attachment angle θ_{an} and exit lip radius y_{en} remain free to vary. This constitutes an optimization in two independent parameters, which is carried out until convergence is attained. This two-dimensional optimization yields a good initial estimate for the final nozzle-base-boattail maximum thrust contour.

The problem is then redefined to be an optimization in three independent variables, with the nozzle and boattail exit radii no longer related. The three-dimensional optimization is then carried out until the maximum thrust producing contour is obtained.

Two special cases may arise that reduce the optimization problem from a three-dimensional problem to either a two-dimensional problem or a one-dimensional problem. The first case arises when an optimization step requires the boattail exit radius y_{eb} to become less than the nozzle exit radius y_{en} (or less than $y_{en} + \Delta y_b$, where Δy_b is the minimum allowable base width). In that case, the boattail exit radius y_{eb} is specified as $y_{eb} = y_{en} + \Delta y_b$, and the solution continues as a two-dimensional optimization problem with θ_{an} and y_{en} as the independent variables. The second case arises when an optimization step requires a nozzle exit radius y_{en} greater than the maximum radius of the boattail. In that case, y_{en} (or $y_{en} + \Delta y_b$) and y_{eb} are set equal to the maximum boattail radius, and the solution continues as a one-dimensional optimization problem with θ_{an} as the single independent variable. The program can efficiently perform an axial iteration optimization in one-dimension, as encountered in a nozzle optimization with fixed exit radius, boattail

optimization with fixed exit radius, or nozzle-base-boattail assembly optimization with fixed boattail contour and nozzle exit radius.

The analysis program is capable of calculating the thrust for nozzle-base-boattail assemblies that have crossing exit radii. That situation can not occur physically, but may occur during the calculation of the function derivatives required to compute an optimization step.

A parametric study of the thrust as a function of the wall geometry for a nozzle-base-boattail assembly was conducted to gain insight into the design of maximum thrust nozzle-base-boattail contours.

The study was conducted for a nozzle having an upstream radius of curvature ρ_{tu} = 3.0 in., a downstream radius of curvature ρ_{td} = 0.5 in., a throat radius y_{tn} = 1.0 in., and a length x_{en} = 10.0 in. The properties of the gas flowing in the nozzle were specific heat ratio γ = 1.2, gas constant R = 60.0 (ft-lbf)/(lbm-R), stagnation temperature T = 6000 R, stagnation pressure P = 1000 psia, and ambient pressure p_{amb} = 0.0. The boattail had a length x_{eb} = 10.0 in., an initial radius y_{tb} = 9.0 in., and an initial expansion contour radius ρ_{td} = 5.0 in. The properties of the air flowing around the boattail were specific heat ratio γ = 1.4, gas constant R = 53.3 (ft-lbf)/(lbm-R), stagnation temperature T = 600 R, and stagnation pressure P = 12.0 psia. Lines of constant thrust were determined for the nozzles and the boattails (see Figs. 3 and 4). It is seen that the thrust varies smoothly, and that there is one clearly defined global maximum thrust for the nozzle. A difference table reduction of the data showed that locally both the nozzle thrust and the boattail thrust varied approximately quadratically.

Nonlinear programming optimization techniques may be divided into two broad categories; derviative-free methods and derivative methods. Derivative-free methods employ systematic searches through the domain of the objective function, in this case the total thrust of the nozzle-base-boattail assembly. These methods seek the extremum by comparing several function values (i.e., the thrust) for various combinations of the independent variables (θ_{an}, y_{en}, y_{eb}), moving to the point having the greater thrust. The idea is to step to the absolute extremum in an organized fashion, without recalculating the objective function at the same point more than once, and without sampling the objective function prohibitively often. This class of methods is most useful when the objective function is a function of many variables.

Derivative methods employ more information about the function than a simple comparison of the function value at two points. They use derivative information about the objective function, and are most effectively employed where the objective function is smooth and depends on a few variables. This class of methods was the more successful in the present investigation.

The general procedure of any optimization method is to begin with a starting point (base point). The algorithm then selects a direction

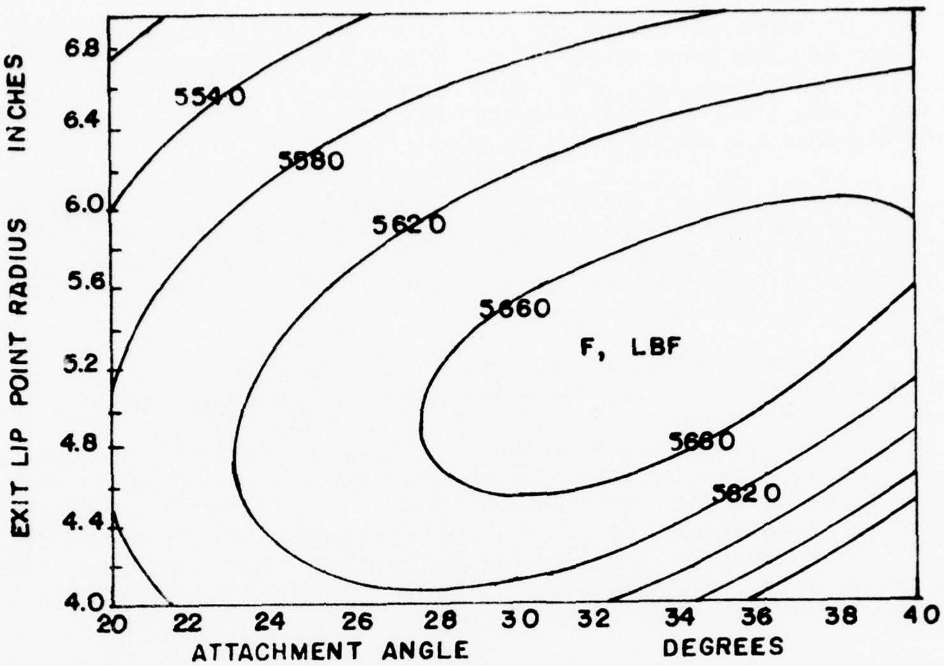

Figure 3. Nozzle Thrust as a Function of Wall Geometry

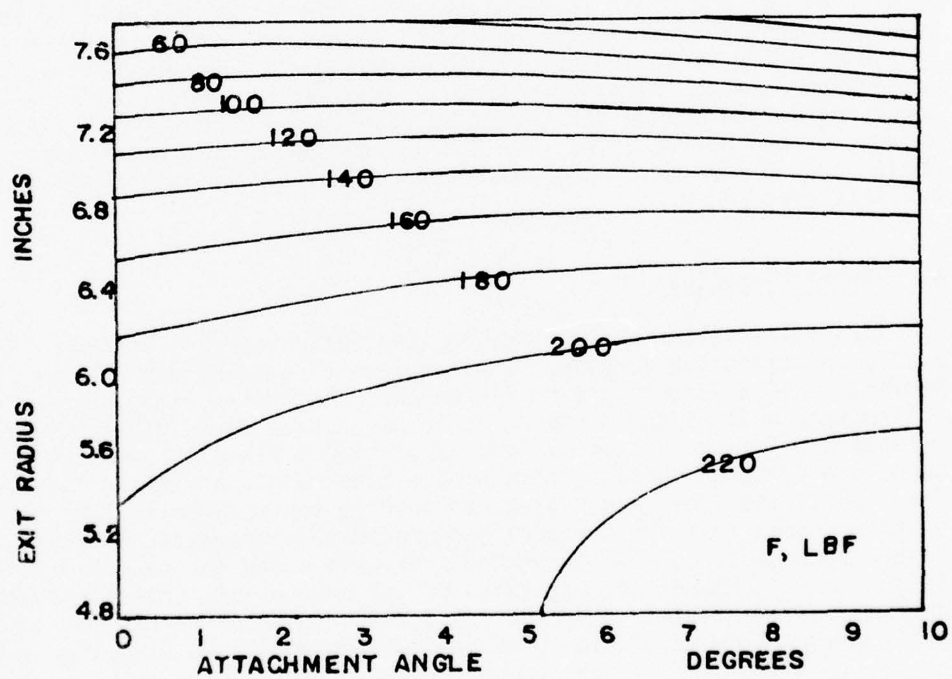

Figure 4. Boattail Thrust as a Function of Wall Geometry

and a distance for moving the base point toward the extremum. The number of base point moves and the number of function evaluations required for each base point move determines the efficiency of the algorithm, since a function evaluation requires much more computational effort than the optimization logic.

Inherent in all optimization procedures is a weakness at abrupt folds of the objective function surface and along ridges and plateaus. Fortunately, the objective function considered in the present investigation (i.e., the thrust) does not have any of these features, thus simplifying the algorithms.

Each optimization method requires an initial estimate of the maximum thrust contour. The method then systematically revises the estimate, moving to the extremum of the objective function. This initial estimate should be reasonably close to the final solution, so that the optimization does not require a prohibitive number of steps, or the analysis does not encounter flow separation or strong internal oblique shock waves. One-dimensional gas dynamic relations for the area ratio required to expand the nozzle flow to the freestream pressure are sufficient to estimate the nozzle exit radius. Some suitably small annular base width specifies the initial boattail exit radius.

Finite perturbations for each of the independent parameters are used in forming the finite-difference derivative approximations used in each of the optimization methods. These perturbations must be large enough so that the corresponding thrust increments are not influenced by the accuracy of the numerical algorithm, yet small enough to accurately approximate local derivatives of the thrust. Also, each perturbation should change the thrust by roughly the same amount. Typical values for the perturbations are 2.0 deg for the nozzle attachment angle, and 20 percent of the nozzle throat radius for the nozzle and boattail exit radii.

A. Axial Iteration

Axial iteration is a multidimensional line search algorithm. It treats the objective function as a function of several coordinates, then performs a line search in each of the coordinate directions successively. A line search finds the distance for a base point move, given a search direction. This may be envisioned as passing a plane through the objective function surface, through the base point, and along the search direction. The line search seeks to move to the maximum on this slice. Given a priori that the surface is approximately quadratic locally, the line search perturbs the independent variable along the search direction by an equal increment on both sides of the base point. First and second-derivative information is formed using these function values for base point moves in one-dimension (see Newton's method), the dimension successively being each of the independent parameters. The new base point estimate is then found, and the objective function is evaluated at that

point. The base point moves to the maximum of the points thus far evaluated, and proceeds to the next independent variable. Convergence is attained when a base point move changes the objective function less than a specified relative tolerance.

Axial iteration requires 3n (where n is the number of independent parameters) function evaluations per pass through the algorithm, and does not have difficulty with poor initial estimates for the maximum thrust contour. Further, this method has one of the good features of the nonderivative methods, in that it steps to the maximum of the points evaluated, not necessarily the predicted point. It is a very simple method, and converges very rapidly.

B. Method of Steepest Descent

Method of steepest descent, or hill climbing, is a first derivative method. By perturbing the objective function n times (where n is the number of independent variables) about the base point (n + 1 function evaluations), a finite difference approximation to the gradient may be calculated. The search direction is then specified along the gradient direction. The algorithm marches stepwise in that direction, seeking the extremum, and moves the base point to that extremum.

If the first step after computing an approximation to the gradient does not increase the objective function, which normally occurs near the maximum, the perturbation sizes and the step size are halved. A new approximation to the gradient is then computed. The procedure is repeated from successive base points until convergence is achieved. This method is relatively simple, and convergence is monotonic.

The principal drawback to the method of steepest descent is that the search can sometimes zig-zag back and forth in a so-called "hem-stitching pattern" (9). This is especially true along ridges in the objective function surface. Consequently, the method may be very inefficient and require a large number of steps. However, the algorithm will eventually converge to the extremum.

This method is recommended primarily in cases where a good initial estimate of the wall geometry is not available or to check the result obtained by any other method to ensure that the absolute maximum is attained. After several base point moves, one of the other, faster converging, methods may be employed to complete the solution.

C. Newton's Method

Newton's method is a second-derivative method. The objective function is approximated by a Taylor series expansion about the base

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(9) Y. Bard, Nonlinear Parameter Estimation, Academic Press, New York, 1974.

point which is truncated after the second-derivative terms. In one-dimension, the following equation is obtained.

$$f(x) = f(a) + f_x(a)(x-a) + \frac{1}{2!} f_{xx}(a)(x-a)^2 \tag{8}$$

where f is the objective function, a is the base point, and the derivatives are evaluated at the base point, a. Taking the derivative of equation (8) with respect to x and noting that f(a), $f_x(a)$, and $f_{xx}(a)$ are constant yields

$$f_x(x) = f_x(a) + f_{xx}(a)(x-a) \tag{9}$$

At a stationary point x = x* (maximum, minimum, or saddle point), $f_x(x^*)$ is identically zero. Consequently, at a stationary point, equation (9) becomes

$$0 = f_x(a) + f_{xx}(a)(x^*-a) \tag{10}$$

Solving equation (10) for x* yields

$$x^* = a - f_x(a)/f_{xx}(a) \tag{11}$$

If f(x) is not quadratic, x* is not exactly the extremum. Usually, however, it is an improvement, and convergence is approached iteratively.

In the case of more than one dimension, x*, a, and $f_x(a)$ are replaced by column vectors, and $f_{xx}(a)$ is an n × n matrix of second derivatives, termed the Hessian. Thus, equation (11) becomes

$$\vec{x}^* = \vec{a} - \vec{f}_{xx}(\vec{a})^{-1} \vec{f}_x(\vec{a}) \tag{12}$$

Solving for $\vec{x}^*$ in equation (12) requires inversion of the Hessian, which, in general, may present a numerical accuracy problem. In the present problem, where $n \leqq 3$, the Hessian has proven to be nonsingular and well conditioned, and no problem arises.

Newton's method converges very rapidly. In fact, it converges in exactly one pass for a quadratic function. However, it does require a relatively large number of function evaluations for each base point move [$(n^2 + 3n + 2)/2$ evaluations/move].

When applying Newton's method, it may be possible to employ the Hessian computed for a given base point for several base point moves before recomputing second derivatives. In the case of a nearly quadratic function, the second derivatives are approximately constant, and the procedure may be useful. The thrust function considered in the present study was not close enough to a quadratic function globally, and the method did not converge unless second derivatives were computed at each step.

Further, when the objective function is not globally quadratic, Newton's method may predict bad base point moves, particularly if the initial estimate is far from the extremum. Hence, there is special incentive for a good first guess.

Newton's method performs quite well in the present problem. Bard (9) states that it is relatively common to find cases where Newton's method may be 25,000 times more efficient than the method of steepest descent! Though not outperforming the other methods nearly so dramatically in the present study, it is clearly an efficient method.

SECTION III

RESULTS

The optimization methods presented in Section II have been applied to the design of maximum thrust nozzle contours, boattail contours, and nozzle-base-boattail contours. These contours are modeled as first- and second-order polynomials. The nozzle thrust predicted by the maximum thrust-producing second-order polynomial is comparable to that predicted by the calculus of variations, which places no restrictions on the functional form of the wall contour. A study comparing the results of the two approaches was conducted for different length nozzles operating at two ambient pressures.

A method of nozzle optimization employing the calculus of variations is based on forming a functional consisting of the following terms:

(1) the term to be maximized (the axial pressure forces along the contour $y = y(x)$),

(2) the design constraint along the nozzle wall $y = y_w(x)$ (such as constant nozzle length or surface area),

(3) the constraint that the contour $y = y_w(x)$ is a streamline, and

(4) the constraint that the governing partial differential equations must be satisfied in the region of the flow field influenced by the nozzle contour.

Each of the constraints is multiplied by a Lagrange multiplier, and a sum of terms (1) through (4) is formed. No constraint is placed on the form of $y = y_w(x)$.

Following the formalism of the calculus of variations, the first variation of the functional is determined and set equal to zero. A set of conditions, known as Euler equations, transversality equations, and corner conditions, results from setting the first variation equal to zero. These design equations guarantee a maximum thrust contour for the assumed constraints. Because the design equations are nonlinear, an iterative solution is performed. A trial contour, $y = y_w(x)$, is assumed, the flow field is determined by the method of characteristics, and the design equations are solved for the Lagrange multipliers. During the above procedure, one transversality equation along the nozzle wall is not employed. That design equation, expressed in terms of the flow field properties, the nozzle contour $y = y_w(x)$, and the Lagrange multi-

pliers, is then checked to see if it is satisfied. If it is satisfied, the assumed contour is the maximum thrust nozzle contour. If it is not satisfied, the contour is modified and the entire procedure is repeated for the modified contour. This relaxation of the nozzle contour is repeated until the design condition is satisfied within a specified tolerance. Typically five to ten modifications of the nozzle contour are required. For each modification, the nozzle flow field and Lagrange multiplier field must be calculated; the Lagrange multiplier field calculation requiring approximately 60 percent as much computation time as that required for the nozzle flow field calculation. Thus, the indirect approach requires from eight to sixteen times the computation time required for one nozzle flow field calculation.

The calculus of variations is an indirect approach to finding maximum thrust nozzle, boattail, and nozzle-base-boattail contours. The advantage of the indirect approach is that no restrictions are placed on the general forms of the contours $y = y_W(x)$. The contours are free to assume any shape required to yield maximum thrust. The disadvantages are that a tremendous amount of time is required to formulate and solve the variational problem to obtain the design equations, and to develop a computer program to solve for the Lagrange multiplier field for a maximum thrust nozzle. This problem is further complicated by consideration of base and boattail effects. Even if these could be accounted for, a subject of some speculation, the entire problem would need to be entirely reworked if changes were made to the gas dynamic model, such as the inclusion of boundary layer drag or nonequilibrium chemistry effects. In spite of these disadvantages, the indirect method is of great value in that it yields absolutely the best solution to the specified problem.

The concept of modeling the contours of a complete nozzle-base-boattail assembly as polynomials, and performing a direct optimization to find maximum thrust has proven to be practical, and an example is presented to demonstrate the application of the optimization techniques.

1. Nozzle Design

Sample nozzle optimizations were carried out using the three methods outlined in Section II for the nozzle illustrated in Fig. 3. Their paths to the extremum is shown in Fig. 5, which illustrates that each of the techniques has the following properties.

a. Each converges.

b. Each converges to the same stationary point.

c. The stationary point converged upon is the global maximum.

d. Each method converges to the global maximum efficiently, without an excessive number of base point moves.

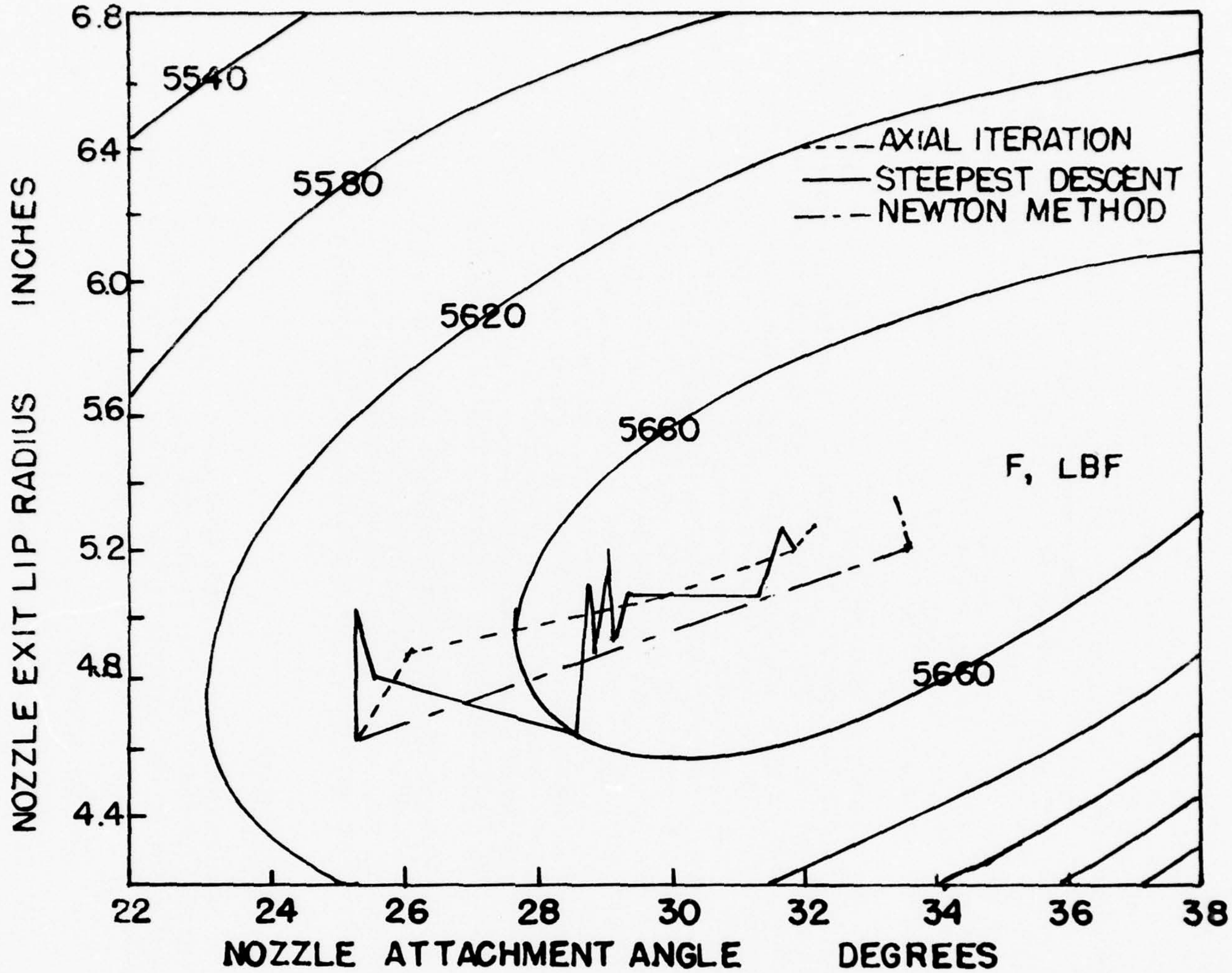


Figure 5. Results of Nozzle Optimization Using Various Procedures

Thus, several practical optimization techniques are available to find the nozzle wall geometry that yields maximum thrust.

To demonstrate that the quadratic wall contours develop thrust comparable to that developed by the calculus of variations contours, a parametric study was performed for a variety of nozzle lengths and two ambient pressures. The results of this study are presented in Table 1. The calculus of variations procedure employed in the comparison is that developed by Rao (10).

The nozzle parameters for the study are upstream radius of curvature ρ_{tu} = 3.0 in., downstream radius of curvature ρ_{td} = 0.5 in., throat radius y_t = 1.0 in., specific heat ratio γ = 1.2, gas constant R = 60 (ft-lbf)/(lbm-R), stagnation temperature T = 6000 R, and stagnation pressure P = 1000 psia.

The thrusts delivered by the contours designed by the calculus of variations are compared with the thrusts delivered by the contours designed by the direct optimization of the second-order polynomial nozzle wall. Both methods predicted essentially the same maximum obtainable thrust, with the direct optimization method not having the analytical difficulties of the calculus of variations approach. Slight variations of the maximum thrust predicted by the two approaches are acceptable because they use different, although similar, flow field analysis programs, and the direct optimizations were terminated by nonzero relative tolerances.

Overall agreement between the two approaches, that is, the calculus of variations and the direct optimization of a second-order polynomial wall nozzle, is very good. For the case of zero ambient pressure, agreement for all cases is within 0.2 percent, which is the approximate numerical resolution of the method of characteristic flow field calculation algorithm. The results for an ambient pressure of 5.0 psia are less compatible, in most part due to the second-order polynomial nozzle wall approximation becoming less good. Thus second-order polynomials effectively approximate maximum thrust nozzle contours, justifying this approach.

2. Nozzle-Base-Boattail Design

The program is capable of computing the design of a nozzle-base-boattail assembly which yields maximum thrust. This is of great practical value in that it dictates the geometry which can produce maximum performance, without necessitating excessive experience or experimental testing. The optimization models the assembly as having a second-order polynomial nozzle contour, a first-order polynomial (conical) boattail contour, and an annular base. This geometry is uniquely specified

(10) G.V.R. Rao, "Exhaust Nozzle Contour For Optimum Thrust," Jet Propulsion, Vol. 28, 1958, pp. 377-382.

TABLE 1. COMPARISON OF THE PERFORMANCE OF MAXIMUM THRUST NOZZLE CONTOURS

		Thrust, lbf		
Nozzle Length (in.)	Ambient Pressure (psia)	Calculus of Variations Prediction	Second-Order Polynomial Wall	ΔF, percent
1.5475	0.0	4556.0	4555.7	0.01
2.3215	0.0	4802.7	4798.1	0.10
3.0706	0.0	4981.4	4979.0	0.05
3.8205	0.0	5124.5	5120.9	0.07
4.5731	0.0	5241.7	5234.9	0.13
6.2965	0.0	5433.1	5428.7	0.08
7.9759	0.0	5569.6	5561.2	0.15
9.6527	0.0	5669.4	5662.5	0.12
12.746	0.0	5806.7	5801.2	0.09
16.9164	0.0	5935.7	5926.6	0.15
20.5326	0.0	6014.4	6003.2	0.19
24.4965	0.0	6081.4	6068.2	0.21
1.4139	5.0	4468.4	4453.1	0.34
2.8861	5.0	4880.4	4848.3	0.66
7.9850	5.0	5311.8	5300.1	0.22
11.836	5.0	5379.0	5367.4	0.22
16.060	5.0	5394.9	5377.6	0.32

knowing nozzle and boattail lengths, initial expansion contours, nozzle attachment angle θ_{an}, nozzle exit radius y_{en}, and boattail exit radius y_{eb}. The last three parameters are varied in such a way as to find that unique combination (θ_{an}, y_{en}, y_{eb}) which yields maximum thrust.

There are several constraints on these variables, besides the minimum and maximum values for each necessary to bracket the extremum. The nozzle and boattail exits must not cross, a physical impossibility. This has been taken one step further in that they can be specified to be at least some specified minimum base width (Δy_b) away from each other. If the optimization violates this, the two exit radii become a fixed distance apart, and the optimization proceeds with one less independent variable ($n = 2$). The nozzle exit radius may also be constrained to be less than some maximum value. This could be encountered if the nozzle exit radius became larger than that of the airframe. This causes the nozzle exit radius to become fixed and the optimization proceeds with one less independent variable ($n = 1$).

A nozzle-base-boattail optimization is presented in Table 2 to illustrate the application of the program. Each of the three methods in Section II is applied to the same case. The nozzle is the same as illustrated in Fig. 3. The boattail is the same as illustrated in Fig. 4 except that it has an outer radius $y_{tb} = 5.3$ in. Note that axial iteration and Newton's method converge comparably, while the method of steepest descent converges to a larger tolerance much more slowly. Newton's method is more prone to failure than axial iteration, because it requires a better estimate of the partial derivatives to take a successful step.

Thus, maximum thrust nozzle-base-boattail contours may be approximated as polynomials and may be directly optimized efficiently using several techniques. The order of the approximating polynomials seems to be effective. The nozzle contour has been shown conclusively to be well approximated by a second-order polynomial. Since the boattail typically contributes an order of magnitude less thrust than does the nozzle, it was modeled as conical. Thus, there are as many as three independent parameters, since the attachment angle may be inferred from the exit lip point for a conical boattail.

TABLE 2. RESULTS OF A NOZZLE-BASE-BOATTAIL THREE-DIMENSIONAL OPTIMIZATION

Method	Step	n	Function Evaluations	θ_{an}, deg	y_{en}, in.	θ_{ab}, deg	y_{eb}, in.	F, lbf
Axial iteration	0	2	1	30.000	4.00	7.059	4.100	5710.71
	1	2	7	27.223	4.296	5.344	4.396	5734.72
	2	2	13	29.365	4.377	4.906	4.477	5737.39
	3	2	19	29.579	4.391	4.722	4.491	5737.46
	0	3	20	29.578	4.391	4.130	4.591	5737.65
	1	3	29	28.700	4.369	3.780	4.650	5737.74
Steepest descent	0	2	1	30.000	4.000	7.059	4.100	5710.71
	1	2	9	29.947	4.497	4.094	4.597	5735.93
	2	2	12	29.947	4.497	4.094	4.597	5735.93
	3	2	16	29.938	4.448	4.385	4.548	5736.09
	4	2	19	29.932	4.448	4.395	4.548	5736.09
	5	2	24	29.888	4.443	4.413	4.543	5736.70
	0	3	25	29.888	4.443	3.923	4.643	5736.27
	1	3	30	29.888	4.443	3.823	4.643	5736.27
	2	3	34	29.888	4.443	3.823	4.643	5736.27
	3	3	39	29.973	4.427	3.889	4.632	5736.29
Newton's method	0	2	1	30.000	4.0	7.059	4.100	5710.71
	1	2	7	29.597	4.235	5.653	4.335	5734.89
	2	2	13	29.552	4.349	4.974	4.449	5737.30
	3	2	19	28.710	4.399	4.674	4.499	5737.47
	0	3	20	28.710	4.399	4.082	4.599	5737.64
	1	3	30	28.551	4.347	3.661	4.670	5737.76

SECTION IV

CONCLUSIONS

A method has been developed for the design of maximum thrust nozzles, boattails, and nozzle-base-boattail contours. These contours may be quickly and efficiently found using any of several direct optimization techniques, providing a practical and dependable basis for design.

Nozzle and boattail optimizations select second-order polynomial wall contours. Nozzle-base-boattail optimizations assume a first-order polynomial (conical) boattail contour and a second-order polynomial nozzle contour. Better approximations for both nozzle and boattail maximum thrust contours may be obtained using higher-order polynomial walls, and hence degrees of freedom. Each additional degree of freedom allows a better fit of the general surface which yields maximum thrust. However, each additional degree of freedom also increases the number of independent contour parameters required to uniquely specify the contour geometry, geometrically increasing the number of function evaluations necessary to perform an optimization. The second-order polynomial wall nozzle and conical wall boattail were selected as an effective compromise. The polynomials are shown to yield substantially identical thrust to that produced by maximum thrust contours designed by the calculus of variations for unconstrained wall geometry.

Three techniques were developed for determining the wall polynomial coefficients. The first method is an axial iteration. It is a fast converging method, and has the least difficulty with a poor initial estimate for the solutions of the methods studied. This method also can take steps from simple sampling, if terms of higher than second order dominate the objective function and second and first derivative information yields a poor step.

The second technique is the method of steepest descent. This method converges monotonically to the extremum and cannot overstep it. It may, however, require an exhorbinant number of steps to attain the maximum. It can step directly away from a poor initial estimate of the maximum, or converge to the extremum from an approximate solution obtained from one of the other methods. This algorithm is primarily intended as a classical reference.

The third method is Newton's method. This, too, is one of the classical optimization methods, and worked very well in the present study. Newton's method does require accurate derivative approximations, and care must be taken to be sure finite-difference increments are large enough to be beyond the numerical resolution of the analysis, yet small enough to detect local functional trends.

All three optimization techniques efficiently select the wall geometry that produces maximum thrust in a nozzle-base-boattail assembly. This makes for a simple and practical approach to the design of maximum thrust nozzle-base-boattail contours.

APPENDICES

APPENDIX A. CHARACTERISTIC AND COMPATIBILITY EQUATIONS

The following equations govern steady two-dimensional isentropic (i.e., rotational) flow,

$$\rho u_x + \rho v_y + u\rho_x + v\rho_y + \delta\rho v/y = 0 \tag{A-1}$$

$$\rho u u_x + \rho v u_y + p_x = 0 \tag{A-2}$$

$$\rho u v_x + \rho v v_y + p_y = 0 \tag{A-3}$$

$$u p_x + v p_y - a^2 u\rho_x - a^2 v\rho_y = 0 \tag{A-4}$$

where $\delta = 0$ for planar flows and $\delta = 1$ for axisymmetric flows. For isentropic flow, the speed of sound, a, is a known function of the static pressure p and the density ρ. Thus,

$$a = a(p,\rho) \tag{A-5}$$

Equations A-1 through A-4 comprise a system of four quasi-linear nonhomogeneous first-order partial differential equations for the four variables u, v, p, and ρ.

This system of partial differential equations may be replaced by an equivalent system of four compatibility equations which are valid along three distinct characteristic curves: the streamline and the two Mach lines. The compatibility equations are total differential equations, as are the equations defining the characteristic curves. Thus, a much simpler numerical method can be employed to solve the system of total differential equations.

The characteristic and compatibility equations for the above system of equations are found by multiplying equations A-1 through A-4 by the unknown parameters σ_1 through σ_4, respectively, and summing. Thus,

$$\sigma_1(A\text{-}1) + \sigma_2(A\text{-}2) + \sigma_3(A\text{-}3) + \sigma_4(A\text{-}4) = 0 \tag{A-6}$$

After this sum is formed, the coefficients of the x derivatives of u, v, p, and ρ are factored out to yield

$$(\rho\sigma_1 + \rho u\sigma_2)\left[u_x + \frac{\rho v\sigma_2}{\rho\sigma_1 + \rho u\sigma_2} u_y\right] + (\rho u\sigma_3)\left[v_x + \frac{\rho\sigma_1 + \rho v\sigma_3}{\rho u\sigma_3} v_y\right] +$$

$$(\sigma_2 + u\sigma_4)\left[p_x + \frac{\sigma_3 + v\sigma_4}{\sigma_2 + u\sigma_4} p_y\right] +$$

$$(u\sigma_1 - a^2 u\sigma_4)\left[\rho_x + \frac{v\sigma_1 - a^2 v\sigma_4}{u\sigma_1 - a^2 u\sigma_4} \rho_y\right] + \sigma_1 \delta\sigma v/y = 0 \qquad \text{(A-7)}$$

The slopes of the characteristic curves, $dy/dx = \lambda$, are then the coefficients of the y derivatives of u, v, p, and ρ in equation A-7. Thus,

$$\lambda = \frac{v\sigma_2}{\sigma_1 + u\sigma_2} = \frac{\sigma_1 + v\sigma_3}{u\sigma_3} = \frac{\sigma_3 + v\sigma_4}{\sigma_2 + u\sigma_4} = \frac{v\sigma_1 - a^2 v\sigma_4}{u\sigma_1 - a^2 u\sigma_4} \qquad \text{(A-8)}$$

If u, v, p, and ρ are assumed to be continuous functions, then $du/dx = u_x + \lambda u_y$, etc., and the terms in brackets in equation A-7 may be replaced by their equivalent values, du/dx, etc. Thus, equation A-7 becomes

$$\rho(\sigma_1 + u\sigma_2)du + \rho u\sigma_3 dv + (\sigma_2 + u\sigma_4)dp +$$

$$u(\sigma_1 - a^2\sigma_4)d\rho + \sigma_1(\delta\rho v/y)dx = 0 \qquad \text{(A-9)}$$

Equation A-9 is the compatibility equation, which is valid along the characteristics defined by Equation A-8. It remains to eliminate σ_1 to σ_4 from equation A-8 and A-9.

Equation A-8, when written as a system of equations considering σ_1 through σ_4 as the unknowns, becomes

$$\sigma_1(\lambda) + \sigma_2(u\lambda - v) + \sigma_3(0) + \sigma_4(0) = 0 \qquad \text{(A-10)}$$

$$\sigma_1(-1) + \sigma_2(0) + \sigma_3(u\lambda - v) + \sigma_4(0) = 0 \qquad \text{(A-11)}$$

$$\sigma_1(0) + \sigma_2(\lambda) + \sigma_3(-1) + \sigma_4(u\lambda - v) = 0 \qquad \text{(A-12)}$$

$$\sigma_1(u\lambda - v) + \sigma_2(0) + \sigma_3(0) + \sigma_4[-a^2(u\lambda - v)] = 0 \qquad \text{(A-13)}$$

For equations A-10 through A-13 to have any solution for the σ's other than the trivial solution of zero, the determinant of the coefficient matrix of equations A-10 to A-13 must be zero. Defining $S = (u\lambda - v)$, that condition yields

$$\begin{vmatrix} \lambda & S & 0 & 0 \\ -1 & 0 & S & 0 \\ 0 & \lambda & -1 & S \\ S & 0 & 0 & -a^2S \end{vmatrix} = 0 \qquad \text{(A-14)}$$

Expanding the determinant gives

$$S^2[S^2 - a^2(1 + \lambda^2)] = 0 \qquad \text{(A-15)}$$

Equation A-15 is a fourth-order polynomial equation in terms of λ; hence four roots, and thus four characteristic directions, should be found. Two roots are obtained by setting the leading term, S^2, equal to zero. Thus,

$$\lambda_o = \left(\frac{dy}{dx}\right)_o = \frac{v}{u} \text{ (repeated twice)} \qquad \text{(A-16)}$$

which is a repeated root, and is the differential equation for the streamline. The subscript o is used to denote the streamline. Hence, the streamline is a dual characteristic in rotational flow. The remaining two roots are obtained from the quadratic term in equation A-15, which, when substituting $S = (u\lambda - v)$, becomes

$$(u^2 - a^2)\lambda^2 - 2uv\lambda + (v^2 - a^2) = 0 \qquad \text{(A-17)}$$

Solving equation A-17 for λ gives

$$\lambda_\pm = \left(\frac{dy}{dx}\right)_\pm = \frac{uv \pm a^2\sqrt{M^2-1}}{u^2 - a^2} \qquad \text{(A-18)}$$

where the subscripts $\pm$ correspond to the $\pm$ root of the quadratic equation. Equation A-18 can be simplified by making the substitution $u = V\cos\theta$, $v = V\sin\theta$, $\theta = \tan^{-1}(v/u)$ and $\alpha = \sin^{-1}(1/M)$. The result is

$$\lambda_\pm = \left(\frac{dy}{dx}\right)_\pm = \tan(\theta \pm \alpha) \qquad \text{(A-19)}$$

Equation A-19 is the differential equation of the Mach lines for supersonic flow; hence, the remaining two characteristics for rotational flow are the Mach lines. The + denotes the left-running Mach line and the - denotes the right-running Mach line.

Thus, three distinct characteristics exist through each point in a rotational flow: the streamline and the two Mach lines.

The compatibility equation valid on each characteristic is obtained from Equation A-9 by solving equations A-10 through A-13 for σ_1 through σ_4. Equations A-10 through A-13 may be rewritten as follows.

$$\sigma_2 = -(\lambda/S)\sigma_1 \tag{A-20}$$

$$\sigma_1 = S\sigma_3 \tag{A-21}$$

$$S\sigma_4 = \sigma_3 - \lambda\sigma_2 \tag{A-22}$$

$$a^2 S\sigma_4 = S\sigma_1 \tag{A-23}$$

Along streamlines, $S = (u\lambda - v) = 0$, and equations A-20 through A-23 become

$$\sigma_1\lambda = 0 \tag{A-24}$$

$$\sigma_1 = 0 \tag{A-25}$$

$$\sigma_2\lambda - \sigma_3 = 0 \tag{A-26}$$

$$0 = 0 \tag{A-27}$$

Thus, along streamlines,

$$\sigma_1 = 0 \qquad \text{and} \qquad \sigma_3 = \lambda\sigma_2 \tag{A-28}$$

and σ_2 and σ_4 are unspecified, and hence arbitrary. Substituting equation A-28 into equation A-9 yields

$$\sigma_2[\rho u\ du + \rho v\ dv + dp] + \sigma_4[u\ dp - a^2 u\ d\rho] = 0 \tag{A-29}$$

Since σ_2 and σ_4 are both arbitrary, their coefficients must be identically zero. Thus, along streamlines

$$\rho u\ du + \rho v\ dv + dp = 0 \tag{A-30}$$

$$dp - a^2\ d\rho = 0 \tag{A-31}$$

On the Mach lines, the quadratic factor in equation A-15 is zero, which gives

$$S^2 - a^2(1 + \lambda^2) = 0 \qquad \text{(A-32)}$$

Solving for (S/a^2) from equation A-32 and substituting the result into equation A-23 yields equation A-22, which shows that only one of these equations is independent on Mach lines. Thus, along Mach lines,

$$\sigma_1 = S\sigma_3, \quad \sigma_2 = -\lambda\sigma_3, \quad \text{and} \quad \sigma_4 = (S/a^2)\sigma_3 \qquad \text{(A-33)}$$

and σ_3 remains arbitrary. Substituting equation A-33 into equation A-9 and dividing by σ_3, which is nonzero and arbitrary, yields the compatibility equation valid on Mach lines. Thus,

$$(\rho v)du_{\pm} - (\rho u)dv_{\pm} + [\lambda_{\pm} - u(u\lambda_{\pm} - v)/a^2\,]dp_{\pm} -$$

$$[v(u\lambda_{\pm} - v)/y]dx_{\pm} = 0 \qquad \text{(A-34)}$$

The subscript + indicates that equation A-34 is valid on left-running Mach lines, and the subscript - denotes that equation A-34 is valid on right-running Mach lines.

An alternate and more useful form of equation A-34 is obtained by making the substitutions $u = V\cos\theta$, $v = V\sin\theta$, $\theta = \tan^{-1}(v/u)$, and $\alpha = \sin^{-1}(1/M)$. Thus,

$$\frac{\sqrt{M^2-1}}{\rho V^2}\,dp_{\pm} \mp d\theta_{\pm} + \delta\frac{v}{yVM\cos(\theta \pm \alpha)}\,dx_{\pm} = 0 \qquad \text{(A-35)}$$

where the upper subscripts on dp, $d\theta$, and dx correspond to the upper signs in the terms $\mp\, d\theta$ and $\cos(\theta \pm \alpha)$, and vice versa. Equation A-30 along a streamline becomes

$$\rho V\, dV + dp = 0 \qquad \text{(A-36)}$$

which is Bernoulli's equation.

Thus, three distinct characteristics are obtained, the streamline and the two Mach lines, with two compatibility equations valid on the streamline and one compatibility equation valid on each Mach line, a total of four compatibility equations. This system of characteristic and compatibility equations is sufficient to replace the original system of four partial differential equations.

In the numerical implementation of equation A-35 along the Mach lines, left-running Mach lines can become essentially vertical. In that

case, the term $dx_+/\cos(\theta + \alpha)$ becomes indeterminate. However, from equation A-19,

$$\frac{dx_\pm}{\cos(\theta \pm \alpha)} = \frac{dy_\pm}{\sin(\theta \pm \alpha)} \tag{A-37}$$

Substituting equation A-37 into equation A-35 yields

$$\frac{\sqrt{M^2-1}}{\rho V^2} dp_\pm \mp d\theta_\pm + \delta \frac{v}{yVM\sin(\theta \pm \alpha)} dy_\pm = 0 \tag{A-38}$$

In the numerical algorithms developed during this investigation, equation A-35 is employed along right-running Mach lines [that is, $dy/dx = \tan(\theta - \alpha)$], and equation A-37 is employed along left-running Mach lines [that is, $dy/dx = \tan(\theta + \alpha)$].

APPENDIX B. KLIEGEL'S TRANSONIC SOLUTION

Various techniques have been developed to handle the transonic flow region in convergent-divergent nozzles, from which the method developed by Kliegel (8) was selected. Kliegel employs an expansion in inverse powers of the normalized throat wall radius of curvature $R(R = \rho_u/y_t)$. His method differs from that developed by Hall (9) by the use of a toroidal coordinate system instead of a cylindrical coordinate system. A toroidal coordinate system results in an expansion in $1/(R + 1)$ instead of $1/R$ for the axial and radial velocity components. This expansion allows for the analysis of nozzles having small normalized throat wall radii of curvature, $R < 1$. The solution derived by Kliegel is expressed in the following form.

$$u = 1 + \frac{u_1(r,z)}{(R+1)} + \frac{1}{(R+1)^2}\left[u_1(r,z) + u_2(r,z)\right] + \frac{1}{(R+1)^3}\left[u_1(r,z) + u_2(r,z) + u_3(r,z)\right] + \cdots \tag{B-1}$$

$$v = \left[\frac{\gamma+1}{2(R+1)}\right]^{1/2}\left\{\frac{v_1(r,z)}{R+1} + \frac{1}{(R+1)^2} \times \left[\frac{3}{2}v_1(r,z) + v_2(r,z)\right] + \frac{1}{(R+1)^3} \times \left[\frac{15}{8}v_1(r,z) + \frac{5}{2}v_2(r,z) + v_3(r,z)\right] + \cdots\right\} \tag{B-2}$$

where

$$z = \left[\frac{2(R+1)}{\gamma+1}\right]^{1/2} x/y_t \tag{B-3}$$

$$r = y/y_t \tag{B-4}$$

In Eq. B-3 z is the transformed normalized axial coordinate. In Eq. B-4 r is the normalized radial coordinate. The complete third-order axisymmetric solution derived by Hall and modified by Kliegel is as follows.

$$u_1 = \frac{1}{2} r^2 - \frac{1}{4} + z \tag{B-5}$$

$$v_1 = \frac{1}{4} r^3 - \frac{1}{4} r + rz \tag{B-6}$$

$$u_2 = \frac{2\gamma + 9}{24} r^4 - \frac{4\gamma + 15}{24} r^2 + \frac{10\gamma + 57}{288} + z\left(r^2 - \frac{5}{8}\right) - \frac{2\gamma - 3}{6} z^2 \tag{B-7}$$

$$v_2 = \frac{\gamma + 3}{9} r^5 - \frac{20\gamma + 63}{96} r^3 + \frac{28\gamma + 93}{288} r + z\left[\frac{2\gamma + 9}{6} r^3 - \frac{4\gamma + 15}{12} r\right] + rz^2 \tag{B-8}$$

$$u_3 = \frac{556\ {}^2 + 1737\gamma + 3069}{10368} r^6 - \frac{388\gamma^2 + 1161\gamma + 1881}{2304} r^4 + \frac{304\gamma^2 + 831\gamma + 1242}{1728} r^2 - \frac{2708\gamma^2 + 7839\gamma + 14211}{82944} + z\left[\frac{52\gamma^2 + 518\gamma + 327}{384} r^4 - \frac{52\gamma^2 + 75\gamma + 279}{192} r^2 + \frac{92\gamma^2 + 1808\gamma + 639}{1152}\right] + z^2\left[- \frac{7\gamma - 3}{8} r^2 + \frac{13\gamma - 27}{48}\right] + \frac{4\gamma^2 - 57\gamma + 27}{144} z^3 \tag{B-9}$$

$$v_3 = \frac{6836\gamma^2 + 23031\gamma + 3027}{82944} r^7 - \frac{3380\gamma^2 + 11391\gamma + 15291}{13824} r^5 + \frac{3424\gamma^2 + 11271\gamma + 15228}{13824} r^3 - \frac{7100\gamma^2 + 22311\gamma + 30249}{82944} r + z\left[\frac{556\gamma^2 + 1737\gamma + 3069}{1728} r^5 - \frac{388\gamma^2 + 1161\gamma + 1181}{576} r^3 + \right.$$

$$\frac{304\gamma^2 + 831\gamma + 1242}{864} r \Big] + z^2 \Big[\frac{52\gamma^2 + 51\gamma + 327}{192} r^3 -$$

$$\frac{52\gamma^2 + 75\gamma + 279}{192} r \Big] - z^3 \left[\frac{7\gamma - 3}{12} r \right] \tag{B-10}$$

Kliegel's analysis is valid for irrotational flow of a perfect gas in an axisymmetric circular arc throat.

APPENDIX C. NOMINAL CASE AND CORRELATION FACTORS FOR BASE PRESSURE MODEL

The pressure, p_b, acting on an annular base area depends on the boattail Mach number and static pressure, the nozzle exit Mach number, exit angle and static pressure, the specific heat ratios of the gases, the stagnation temperatures of the two streams, and the ratio of nozzle exit radius to boattail exit radius. A model which accurately predicts the base pressure as a function of all these parameters has not been developed yet, but the model by Addy (12) does include the effects of each of these parameters and gives a reasonable approximation to the base pressure. Byington and Hoffman (1) correlated his data by a curve fit of the form

$$\frac{p_b}{p_{eb}} = f(p_{en}/p_{eb})\eta(M_{eb})\eta(M_{en})\eta(T_{eb}/T_{en})\eta(\theta_{en}) \tag{C-1}$$

The nominal case used for $f(p_{en}/p_{eb})$ was: $M_{eb} = 2.0$, $\gamma = 1.2$, $M_{en} = 2.0$, $\theta_{en} = 0$ deg, $(T_{eb}/T_{en}) = 1.0$, and $(y_{eb}/y_{en}) = 0.6$. This resulted in the following third-order polynomial curve fit.

$$f(p_{en}/p_{eb}) = 0.27435080 + 0.13825822 \times (p_{en}/p_{eb}) - 0.01754651 \times (p_{en}/p_{eb})^2 + 0.00097156 \times (p_{en}/p_{eb})^3 \tag{C-2}$$

The four η functions are correlation factors which are applied to the nominal case. Each factor accounts for variations in the parameter denoted by its argument and was obtained by a curve fit of the data presented by Addy (12). Correlation factors based on the specific heat ratios and exit lip point coordinates have been neglected since their influence on the base pressure is slight. The four η correlation factors are

$$\eta(M_{eb}) = \frac{2.0}{M_{eb}} \tag{C-3}$$

$$\eta(M_{en}) = \frac{3.5 - 2.5/M_{en}}{M_{en}} \tag{C-4}$$

$$\eta(T_{eb}/T_{en}) = 0.978 + 0.022(T_{en}/T_{eb}) \qquad \text{(C-5)}$$

$$\eta(\theta_{en}) = 1.0 + \theta_{en}/45.0 \qquad \text{(C-6)}$$

The correlation factors $\eta(M_{en})$ and $\eta(\theta_{en})$ are not accurate near $p_{en}/p_{eb} = 1.0$.

APPENDIX D. PROGRAM DESCRIPTION

A brief description of the computer program is presented here to supplement the comments in the code. Where appropriate, the logic of each subroutine is discussed. A list of the programs, subroutines, and functions subprograms is presented in Table D-1. A diagram of the control logic followed by the program is shown in Table D-2.

The computer program consists of approximately 4000 cards. It requires 30 second of central processor time for compilation on the CDC 6500 computer. Nozzle flow field analyses typically require 8 to 10 seconds of computation time, although that time is reduced to 4 to 6 seconds when the secondary-start line option is utilized effectively. Boattail flow field analyses are substantially faster than nozzle flow field analyses, requiring only 4 to 6 seconds of computation time.

TABLE D.1. LIST OF PROGRAMS, SUBROUTINES, AND FUNCTION SUBPROGRAMS

MAIN	THERMØ
TITLE	EM
NØZZL	PØINT
BØTTAL	INWALL
NZBSBT	F
ØPTN	DRWALL
ØPTB	WALLPT
ØPTNB	AXIS
CØNBØT	BASE
PUSH	TRAN2D
INPUTT	TRAN1D
DATAN	TRAND4
DATAB	TRAN12
IVLINN	TRAN14
IVLINB	TRAN24
XYLINE	TRAN3D
STARTN	TRANSL
STARTB	AXITER
LØGICN	HILCLM
LØGICB	NEWTØN
WRITE	GAUSS
THRST	

TABLE D.2. LOGICAL SUBROUTINE CALLING SEQUENCES

PROGRAM MAIN

ANALYSIS			OPTIMIZATION		
NOZZL	BOTTAL	NZBSBT	OPTN	OPTB	OPTNB
INPUTT	INPUTT	INPUTT	INPUTT	INPUTT	INPUTT
DATAN	DATAB	DATAN	DATAN	DATAB	DATAN
IVLINN	IVLINB	IVLINN	IVLINN	IVLINB	IVLINN
STARTN	STARTB	STARTN	STARTN		STARTN
LOGICN	LOGICB	LØGICN	LØGICN		LØGICN
		DATAB			DATAB
		IVLINB			IVLINB
		STARTB			
		LØGICB			
		BASE			

AXITER HILCLM NEWTON

PUSH

MAIN. Program MAIN reads the first data card, which specifies whether an analysis or an optimization is to be performed, and the type of geometry. The parameter IØPT specifies either an analysis (IØPT = 1) or an optimization (IØPT = 2). ITYPE specifies the type of geometry: a complete nozzle-base-boattail system (ITYPE = 1), a nozzle only (ITYPE = 2), or a boattail only (ITYPE = 3). The first card also contains an alphanumeric job title for the run. When successive runs are being made, this data card must be specified for each run. Program MAIN then calls the appropriate analysis or optimization master control subroutine.

TITLE. Subroutine TITLE writes out the standard comments and job title on the first page of the output.

NØZZL. Subroutine NØZZL contains the overall logic for the nozzle flow field analysis. All of the input data for the nozzle are read in by calling subroutine INPUTT. Then subroutine IVLINN is called to generate the initial-value line, and subroutine STARTN is called to determine the wall contour parameters. Subroutine LØGICN, which contains the logic for determining the nozzle flow field by the method of characteristics, is then called.

BØTTAL. Subroutine BØTTAL contains the overall logic for the boattail flow field analysis. All of the input data for the boattail are read in by calling subroutine INPUTT. The subroutine IVLINB is called to generate the initial-value line, and subroutine STARTB is called to determine the wall contour parameters. Subroutine LØGICB, which contains the logic for determining the boattail flow field by the method of characteristics, is then called.

NZBSBT. Subroutine NZBSBT contains the overall logic for the analysis of the nozzle-base-boattail combination. There is a section for the nozzle flow field analysis, a section for the boattail flow field analysis, and a section for the base region analysis. The analyses are performed in the order stated. The same logic pattern is followed for the flow field analyses for both the nozzle (see subroutine NØZZL) and the boattail (see subroutine BØTTAL). The logic pattern reads in all of the input data, generates an initial-value line, calculates wall contour parameters, and employs the method of characteristics to analyze the flow field. After the flow fields for both the nozzle and boattail have been computed, subroutine BASE calculates the base pressure and the base thrust.

ØPTN. Subroutine ØPTN is the master logic program for nozzle contour optimization. All of the input data are read in and the initial-value line is generated. Then LØGICN is called to compute the secondary-start line, which is a right-running Mach line attached to the minimum allowed throat angle [ANMIN(1)] and extending either to the axis of symmetry or to

the exit plane of the nozzle. LØGICN then restarts from this line, thus increasing the efficiency of multiple nozzle flow field calculations in a nozzle or a nozzle-base-boattail optimization (see discussion in Section II). The parameter IMETH specifies which optimization algorithm is to be employed (IMETH = 1, Axial Iteration; IMETH = 2, Method of Steepest Descent; IMETH = 3, Newton's Method). The appropriate subroutine is then called to perform the nozzle contour optimization.

ØPTB. Subroutine ØPTB is the master logic program for boattail contour optimization. All of the input data are read in and the initial-value line is generated. The optimization method specified by the parameter IMETH (see subroutine ØPTN) is called to perform the boattail contour optimization.

ØPTNB. Subroutine ØPTNB is the master logic program for a complete nozzle-base-boattail contour optimization. All of the input data are read in and the initial-value line for the nozzle is generated. LØGICN is called to compute the nozzle secondary-start line (see discussion in Section II). The optimization method to be employed is specified by the parameter IMETH (see subroutine ØPTN). The appropriate subroutine is called to perform the combined nozzle-base-boattail optimization.

CØNBØT. Subroutine CØNBØT calculates the attachment angle corresponding to a conical boattail, given the boattail exit radius and initial expansion contour. This subroutine employs a Newton's method zero finding algorithm to iteratively solve for the attachment angle.

PUSH. Function PUSH calls the appropriate subroutines to compute the thrust for the specified geometry using the previously specified start lines for the nozzle and/or the boattail. The wall geometry for a nozzle-base-boattail configuration is specified by the following three independent parameters: (1) the nozzle throat attachment angle θ_{an}, (2) the nozzle exit radius y_{en}, and (3) the boattail exit radius y_{eb} (see Figures E-1 and E-2). Subroutine PUSH sets the variable PUSH equal to the value of the thrust for the specified geometry, and returns to the calling program. This function subprogram serves as the objective function for the optimization algorithms.

INPUTT. Subroutine INPUTT reads in all of the input data through NAMELIST INFØ. If an input variable is not specified on the data cards, then that variable is set equal to its default value given in subroutine INPUTT. A discussion of the input is given in Appendix E.

DATAN. Subroutine DATAN defines all of the variables employed in the method of characteristics calculations to be those corresponding to the nozzle. All of the nozzle input variables are printed out at the beginning of the analysis.

DATAB. Subroutine DATAB defines all of the variables employed in the method of characteristics calculations to be those corresponding to the boattail. All of the boattail input variables are printed out at the beginning of the analysis.

IVLINN. Subroutine IVLINN generates an initial-value line for the nozzle flow field analysis. When the input parameter IVLN = 1, the points on the initial-value line are read in. Each data card must contain the location, velocity magnitude, pressure, flow angle, and density at a point on the initial-value line. The number of data cards must be equal to the parameter NPN. If the parameter IVLN = 0, Kliegel's initial-value line is generated. This line begins on the wall at the minimum cross section of the throat and ends downstream a distance EPSLØN along the axis. EPSLØN is computed in subroutine IVLINN and is a function of the specific heat ratio, throat radius, and throat upstream radius of curvature. All of the properties for each point on the initial-value line are stored in the data storage array SL(K,I) and SLN(K,I).

IVLINB. Subroutine IVLINB determines the initial-value line for the boattail. If the input parameter IVLB = 1, the points on the initial-value line are read in. Each data card must contain the location, velocity magnitude, pressure, flow angle, and density at a point on the initial-value line. The number of data cards must be equal to the parameter NPB. If the parameter IVLB = 0, a uniform flow initial-value line is generated. This uniform line is computed from the free-stream Mach number and the stagnation properties which are specified in the input data. All of the properties for each point on the initial-value line are stored in the data storage arrays SL(K,I) and SLB(K,I).

Each point on the initial-value line is the beginning of a new right-running Mach line. The initial-value line must be specified long enough to insure that the last point on the initial-value line will generate a right-running Mach line beyond the exit lip point of the boattail. For the uniform flow initial-value line, the x-coordinate spacing of the points is determined by the input parameter SPACE. If SPACE = 1.0, the x-coordinate of the last point on the initial-value line will be equal to the boattail length x_{en}, and the last right-running Mach line will always go beyond the wall exit lip point of the boattail. To keep from generating unnecessary points on the initial-value line, SPACE should be specified between 0.3 and 0.6, depending on the freestream Mach number.

XYLINE. Subroutine XYLINE generates geometrically progressively spaced y-coordinates and the corresponding x-coordinates for points along the nozzle initial-value line. These points may be closely spaced next to the nozzle wall and spread apart as they approach the nozzle axis. The geometric progression is determined by the input parameter RATIØI. For example, if RATIØI = 5.0, then the y distance between the last two points on the initial-value line at the axis will be 5.0 times greater than the y distance between the first two points at the wall. This type of

point spacing permits a closer spacing of points near the wall where the flow property gradients are larger. If RATIØI = 1.0, the y-coordinates will be uniformly spaced along the initial-value line.

STARTN. Subroutine STARTN determines the geometric parameters that specify the nozzle contour. The type of wall contour and the parameters that specify the wall contour are printed out. A conical wall, a second-order polynomial wall, or a tabular wall contour may be selected. The input parameter IWALLN determines which type of wall contour is selected. IWALLN = 1 specifies a conical nozzle, IWALLN = 2 specifies a second-order polynomial nozzle, and IWALLN = 5 specifies a tabular nozzle. The half-angle of the conical nozzle is specified by the throat expansion angle θ_{an}. For a second-order polynomial wall, the wall exit lip point is determined by specifying either the exit radius y_{en} or the exit angle θ_{en} (see Figure E-1). Subroutine STARTN computes the coefficients for the conical and second-order polynomial wall contours for the following equation.

$$y(x) = a + bx + cx^2 \tag{D-1}$$

The number of points in a tabular wall is specified by the parameter NWALLN, and the arrays XWN(100) and YWN(100) contain the respective x- and y-coordinates of the NWALLN points. The conical wall option and the tabular wall option may be employed only with the flow field analysis option. For the optimization option, the second-order polynomial wall contour option must be specified.

STARTB. Subroutine STARTB determines the geometric parameters that specify the boattail contour. The type of wall contour and the corressonding parameters are printed out. Four types of boattail contours may be selected: conical, cylindrical, second-order polynomial, or tabular contour. The parameter IWALLB determines which type of wall contour is selected. IWALLB = 1 specifies a conical boattail, IWALLB = 2 specifies a second-order polynomial boattail, IWALLB = 3 specifies a cylindrical boattail, and IWALLB = 5 specifies a tabular boattail. The half-angle of the conical boattail is specified by the initial expansion angle θ_{ab}. For a second-order polynomial wall, the wall exit lip point is determined by specifying either the exit radius y_{eb} or the exit angle θ_{eb}. Subroutine STARTB computes the coefficients for the boattail wall contours employing the following equation.

$$y(x) = a + bx + cx^2 \tag{D-2}$$

The number of points in a tabular wall is specified by NWALLB, and the arrays XWB(100) and YWB(100) contain the respective x- and y-coordinates of the NWALLB points. The tabular wall option may be employed only with the flow field analysis option. For the optimization option, the conical wall contour option must be specified for a complete nozzle-base-boattail configuration. For the optimization of a boattail contour

by itself, the second-order quadratic wall contour option is employed. When a complete nozzle-base-boattail analysis is being conducted, the base pressure model is restricted to boattails which have the same x-coordinate for the wall exit lip point and the nozzle exit lip point. However, if another base pressure model is inserted which includes the effects of different x-coordinates for the exit lip points of the nozzle and boattail, this will in no way affect any other part of the analysis.

LØGICN. Subroutine LØGICN contains the logic which generates the solution network for the nozzle flow field. Left-running Mach lines are followed to produce the characteristic mesh which constitutes the solution network. These left-running Mach lines originate from either the initial-value line or the nozzle axis. LØGICN determines which of these starting points is to be used for each left-running Mach line. In addition to determining the starting point, LØGICN monitors how many interior points are to be calculated, whether an indirect or direct wall point is to be computed, and when to stop the calculations. The following paragraphs give a more detailed description of the mechanics of subroutine LØGICN.

A thorough knowledge of the I-J characteristics coordinate system shown in Fig. D-1 is required to understand the overall solution logic. Each I-J point represents a calculated point in the characteristic network. The I-coordinate lines represent right-running Mach lines, and the J-coordinate lines represent left-running Mach lines. The point (NPN,1) is the first point of the solution network.

After initiating various control variables, the first point in the characteristic network is obtained. As shown in Figure D-1, this point (NPN,1) is located at the wall on the initial-value line.

When the main loop is entered, a decision must be made immediately as to which type of starting point is to be used. The first NPN - 1 times through the loop an initial-value line point is selected. Thereafter, an axis point is selected. This axis point is located at the intersection of the nozzle axis and the right-running Mach line from the second point on the previous left-running Mach line. After obtaining the starting point for the next left-running Mach line, interior points are calculated along the line, and the variable I is incremented by one after each point is calculated. This procedure continues until the I-coordinate of the interior point is equal to ILIMIT, when a wall point must be computed. After the wall point is computed, the nozzle thrust is computed by subroutine THRST.

Two different methods are used to compute a wall point. Along the circular arc initial expansion region of the throat, where property gradients are high, points are specified along the wall through which the left-running Mach lines must pass. Beyond the initial expansion region of the circular arc throat, the left-running Mach lines are ex-

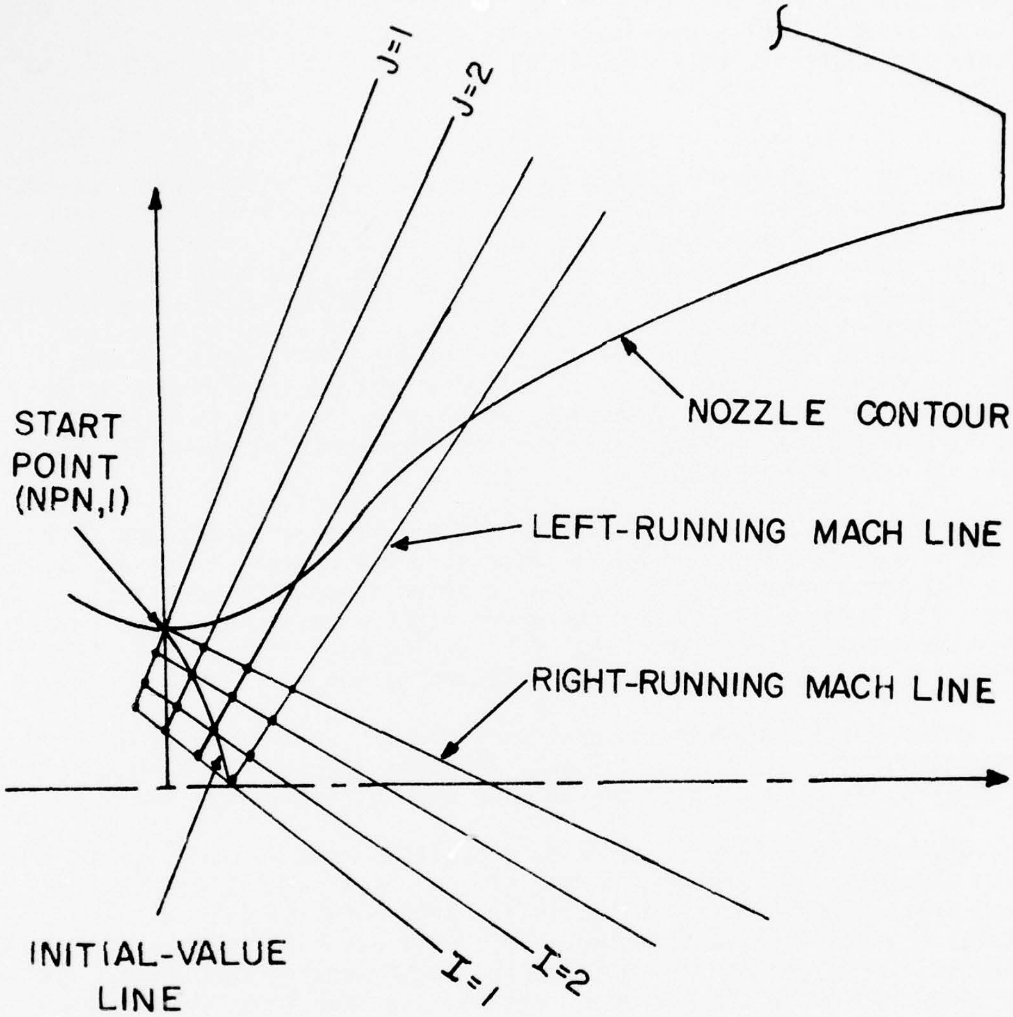


Figure D-1. Characteristic Coordinate System for the Nozzle Flow Field

tended directly to intersect the wall. This latter technique does not require any special modification of the logic, but the former technique does. To handle that situation, the special numbering system shown in Fig. D-2 has been devised. The specified points along the wall are spaced geometrically to increase the accuracy of the solution. The parameter RATIØW specifies the ratio of the angular increment spanned by the last two points on the initial expansion contour adjacent to the throat attachment angle θ_{an} to the angular increment spanned by the first two points at the throat. Knowing the throat attachment θ_{an}, RATIØW, the location of the throat, and the nozzle downstream radius of curvature, the wall point locations may be computed. The interpolated points labeled by 1's on Figure D-2 are not stored. The calculated wall point (I + 1, J - 1) illustrated in Figure D-2(a) is stored in the previous left-running Mach line array at the (I + 1) location. After storing and printing the wall point, the interior point (I + 1, J) illustrated in Figure D-2(b) is calculated. This entire procedure, which is referred to as the inverse wall point scheme, is repeated until a specified wall point requires the x-value of the interpolated point 1 to exceed that of the last interior point. When this occurs, the parameter ITER is set equal to 99 in subroutine INWALL, and the next left-running Mach line is started from either the initial-value line or the axis. The last inverse wall point is computed at the maximum throat angle θ_{an}.

The main loop continues to generate left-running Mach lines by the direct wall point method until the x-coordinate of the wall point exceeds the specified nozzle length x_{en}. Then the inverse wall point routine is called, using the nozzle exit lip point as the specified wall point. This completes the nozzle flow field analysis.

After the calculation of each interior point, a check is made to see if the Mach lines crossed. If the right-running Mach lines crossed, then the point just calculated is dropped from the flow field and a message is printed indicating this has been done. The properties at the next interior point are then based on the next point along the previous left-running Mach line. If the left-running Mach lines cross, the calculation of points along the new left-running Mach line is terminated. The remaining points up to the wall are defined to be those of the previous left-running Mach line which was just crossed. A message is printed indicating that the left-running Mach lines crossed.

Two important storage procedures are carried out in subroutine LØGICN. A secondary-start line, which is employed during the optimization procedure, is stored when the parameter ISSL is non-zero (see Section II). The secondary-start line attaches to the nozzle wall at the minimum allowed throat attachment angle. It is stored in the same location as the initial-value line. Subroutine LØGICN restarts the analysis from this secondary-start line. The value for ISSL = NPN + NPW when the secondary-start line is being stored. When the complete secondary-start line has been determined, the value of ISSL is 0. Note that subsequent restarts must have greater throat attachment angles than the minimum angle which determined the secondary-start line. If the

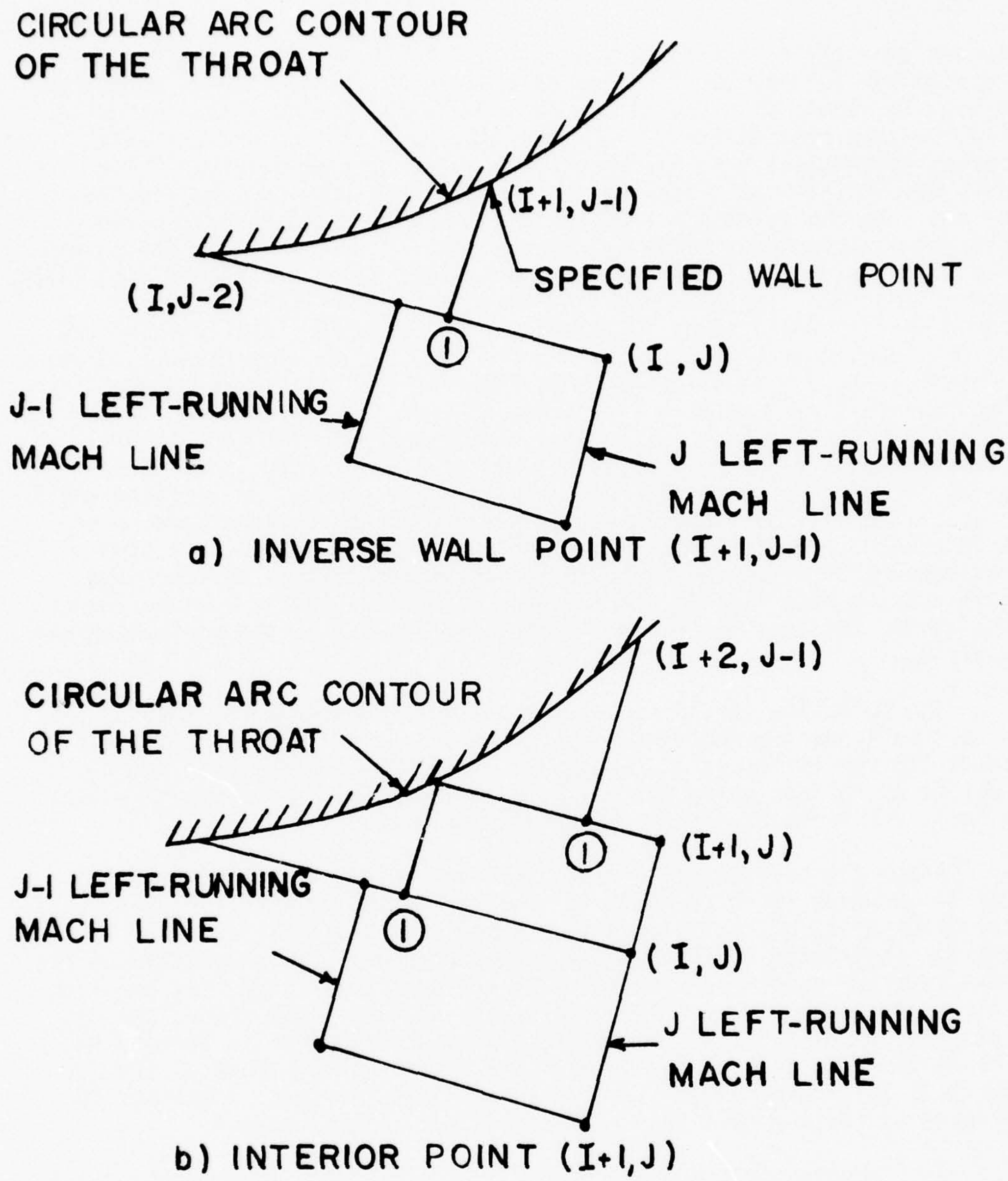


Figure D-2. Characteristic Coordinates for an Inverse Wall Point and an Interior Point

analysis option is utilized, ISSL is set equal to a large number and a secondary-start line is not stored. The flow properties at the final wall point of the flow field are stored in the array DFINAL.

LØGICB. Subroutine LØGICB contains the logic that generates the solution network for the boattail flow field. Right-running Mach lines are followed from the initial-value line to produce the characteristic mesh. Subroutine LØGICB determines how many interior points are to be calculated, when a wall point is to be computed, and when to stop the calculations. The following paragraphs give a detailed description of the mechanics of subroutine LØGICB.

As in the nozzle flow field analysis, an I-J characteristic coordinate system is employed to perform the tracking and storage of the computed points in the boattail flow field. The I-J coordinate system is shown in Fig. D-3. The I-coordinate lines represent right-running Mach lines, and the J-coordinate lines represent left-running Mach lines. The point (1,1) is the first point of the solution network.

The main loop of subroutine LØGICB is exceptionally straightforward since each right-running Mach line must originate from the initial-value line and terminate at the wall. Thus, the first step in the loop is to obtain a point from the initial-value line. Interior point calculations are then performed until the J-coordinate equals JLIMIT. Then a direct wall point is determined, and the thrust associated with that point is computed. The parameter JLIMIT is set equal to J + 1, and the entire loop is repeated. When the x-coordinate of the wall point exceeds the boattail length x_{eb} the inverse wall point routine is called to determine the flow properties at the boattail exit lip point. The thrust is then computed. The final point on the last Mach line is stored in the array DFINAL. This completes the flow field analysis for the boattail.

If the initial-value line for the boattail flow field analysis has been specified too short, the analysis will be contiued until the last point on the initial-value line has been used, at which time the last wall point still does not exceed the specified boattail length x_{en}. When this occurs, an error message will be printed and the program stops.

WRITE. Subroutine WRITE prints out the results of the characteristic calculations and the secondary-start line. It also determines the proper headings and carriage controls necessary for the various print options.

THRST. Subroutine THRST calculates the thrust at a wall point for the nozzle. For comparison, the one-dimensional flow thrust is also computed. This one-dimensional thrust appears on the last line of output for each left-running Mach line in the nozzle. Subroutine THRST also computes the fraction of the thrust and the mass flow rate between an interior point and the axis.

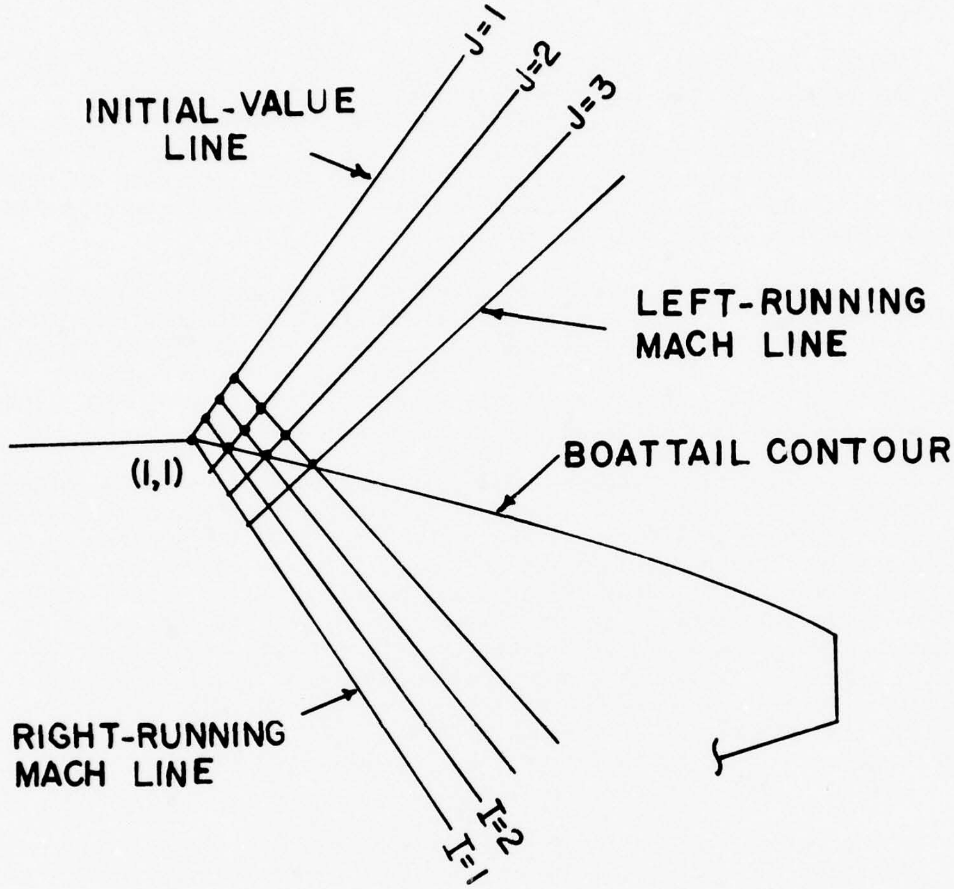


Figure D-3. Characteristic Coordinate System for the Boattail Flow Field

THERMØ. Subroutine THERMØ calculates the thermodynamic properties of the gas which are required by the characteristic and compatibility equations. The Mach number, M, temperature, t, and the speed of sound, a, are computed as a function of the density, ρ, pressure p, and velocity V. The subroutine built into the program is based on the properties of a thermally and calorically perfect gas. Other gas thermodynamic models may be considered by appropriate modifications to subroutine THERMØ.

EM. Function EM determines the Mach number as a function of the area ratio for the one-dimensional flow of a perfect gas. That information is required in subroutine THRST for the one-dimensional thrust comparison. If a gas thermodynamic model other than a perfect gas is specified in subroutine THERMØ, function EM must be modified accordingly.

PØINT. Subroutine PØINT evaluates the solution at an interior point in the flow field using the method of characteristics for rotational flow. The solution process employs the modified Euler predictor-corrector technique for solving the flow field characteristic and compatibility equations. An average property scheme is used to compute the coefficient of each differential. The point labeling scheme for this subroutine is shown in Fig. D-4. The location and flow properties are known at points 1 and 2. Point 4 is located at the intersection of the right-running and left-running Mach lines. The location of point 3 is obtained by extending a streamline back from point 4 until it intersects the line between points 1 and 2. Flow properties at point 3 are obtained by linear interpolation between points 1 and 2. The flow properties p, ρ, V, and θ at point 4 are then obtained by solving the characteristic and compatibility equations. The equations programmed are as follows. Along the right-running Mach line,

$$\frac{dy}{dx} = \tan(\theta - \alpha) \tag{D-3}$$

$$\frac{\sqrt{M^2 - 1}}{\rho V^2} dp + d\theta + \delta \left[\frac{v}{yMV \cos(\theta - \alpha)} \right] dx = 0 \tag{D-4}$$

Along the left-running Mach line,

$$\frac{dy}{dx} = \tan(\theta + \alpha) \tag{D-5}$$

$$\frac{\sqrt{M^2 - 1}}{\rho V^2} dp - d\theta + \delta \left[\frac{v}{yMV \sin(\theta + \alpha)} \right] dy = 0 \tag{D-6}$$

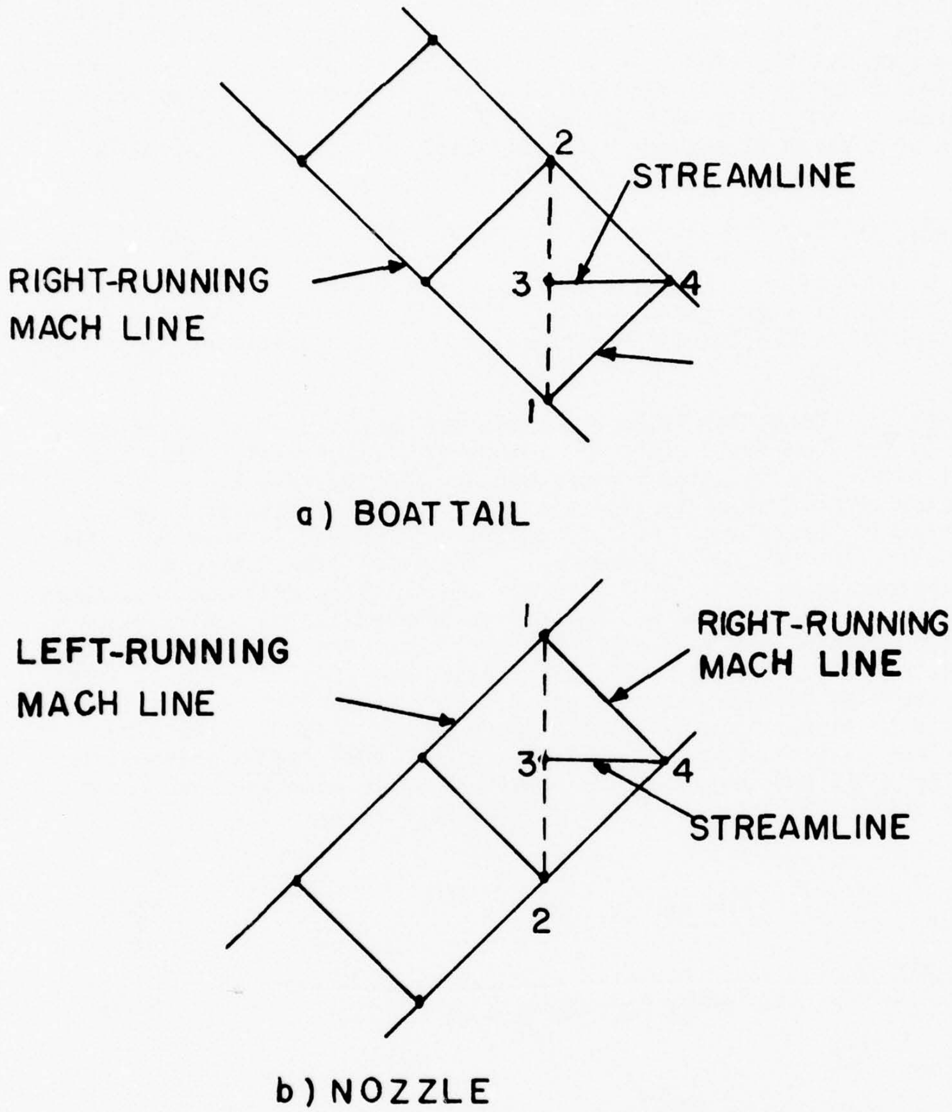


Figure D-4. Point Labeling Scheme for an Interior Point

Along the streamline,

$$\frac{dy}{dx} = \frac{v}{u} \tag{D-7}$$

$$\rho V dV + dp = 0 \tag{D-8}$$

$$dp - a^2 d\rho = 0 \tag{D-9}$$

The modified Euler predictor-corrector technique with average properties is used in the numerical analysis. The remaining flow properties are obtained by calling subroutine THERMØ.

<u>INWALL</u>. Subroutine INWALL evaluates the solution at a specified wall point in the circular arc throat region using an inverse method of characteristics for rotational flow. The location and flow properties are known at points 2, 3, and 5 in Fig. D-5. The solution is sought at point 4 where the location (x_4,y_4) and slope θ_4 are already known. A left-running Mach line is extended rearward from point 4 to point 1, where the flow properties are obtained by linear interpolation between points 2 and 3. The compatibility equations are then solved for the flow properties p, ρ, V, and θ at point 4. The equations programmed are as follows. Along the left-running Mach line,

$$\frac{dy}{dx} = \tan(\theta + \alpha) \tag{D-10}$$

$$\frac{\sqrt{M^2 - 1}}{\rho V^2} dp - d\theta + \delta \left[\frac{v}{yMV \sin(\theta + \alpha)} \right] dy = 0 \tag{D-11}$$

Along the nozzle wall,

$$\frac{dy}{dx} = \frac{v}{u} \tag{D-12}$$

$$\rho V dV + dp = 0 \tag{D-13}$$

$$dp - a^2 d\rho = 0 \tag{D-14}$$

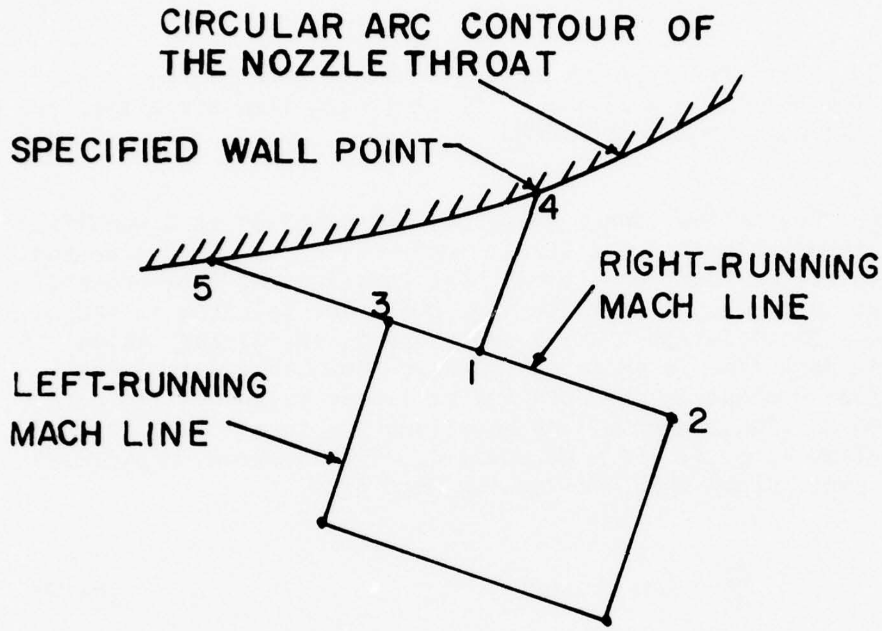


Figure D-5. Point Labeling Scheme for an Inverse Wall Point

The modified Euler predictor-corrector technique with average properties is used in the numerical analysis. The remaining flow properties are obtained by calling subroutine THERMØ.

F. Function F calculates the nozzle wall point spacing required for the portion of the circular arc throat between the secondary-start line attachment point and the circular arc throat attachment point. The objective is to maintain consistent spacing for those points upstream of the secondary-start line attachment point and those downstream of that point.

First, function F computes the number of wall points from the minimum throat attachment point to the specified throat attachment point using the original wall point spacing between the nozzle throat and the minimum throat attachment point. Then an iteration is performed to find the exact spacing which allows the last point to lie at the specified throat attachment point. Thus, the inverse wall points located downstream of the secondary start line point are spaced consistently with those upstream of that point, and the last inverse wall point is the specified throat attachment point.

DRWALL. Subroutine DRWALL determines the solution at a direct wall point using the method of characteristics for rotational flow. The location and flow properties are known at points 2 and 3 in Fig. D-6. Since the wall contour $y(x)$ and the wall slope $dy(x)/dx = \tan\theta$ are known, it is only necessary to locate point 4 on the wall and to calculate the flow properties p, ρ, and V at that point. Left-running characteristics and compatibility equations are used for the nozzle wall, and right-running characteristic and compatibility equations are used for the boattail wall. The equations programmed are as follows. Along the Mach lines,

$$\frac{dy}{dx} = \tan(\theta \pm \alpha) \tag{D-15}$$

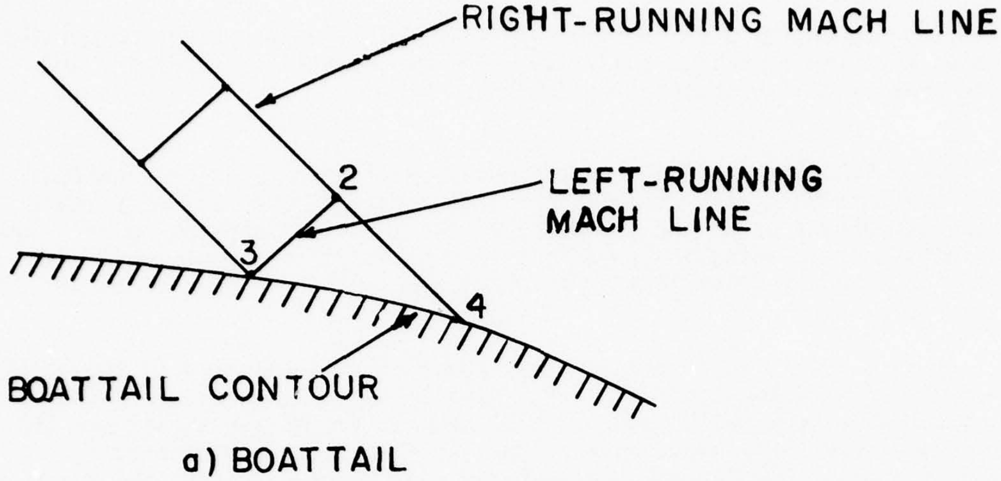
$$\frac{\sqrt{M^2 - 1}}{\rho V^2} dp_{\pm} \mp d\theta_{\pm} + \delta\left[\frac{v}{yMV\sin(\theta \pm \alpha)}\right] dy_{\pm} = 0 \tag{D-16}$$

The upper signs are used for the nozzle analysis, and the lower signs are used for the boattail analysis. Along the walls,

$$\frac{dy}{dx} = \frac{v}{u} \tag{D-17}$$

$$\rho V dV + dp = 0 \tag{D-18}$$

$$dp - a^2 d\rho = 0 \tag{D-19}$$



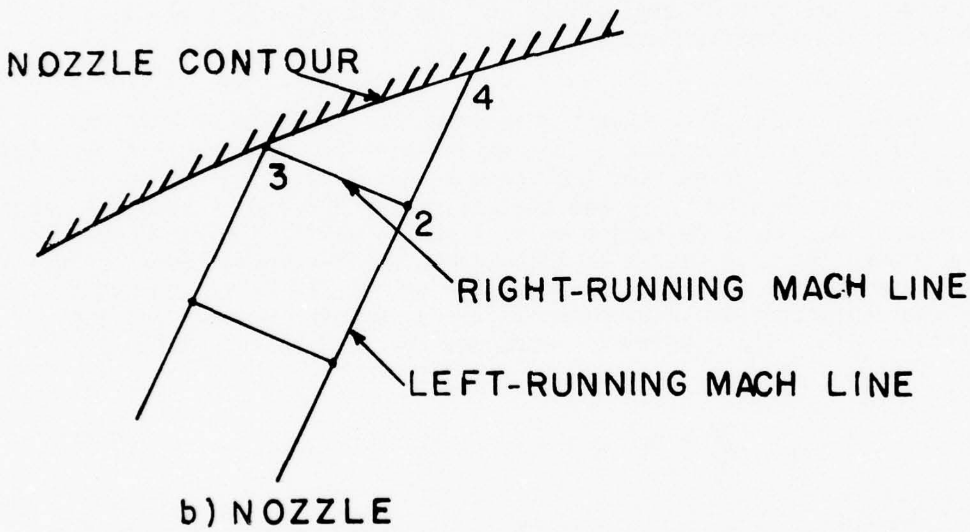


Figure D-6. Point Labeling Scheme for a Direct Wall Point

The modified Euler predictor-corrector technique with average properties is employed in the numerical analysis. The remaining flow properties are obtained by calling subroutine THERMØ.

WALLPT. Subroutine WALLPT locates the intersection of a Mach line with the wall contour. The x-coordinate, y-coordinate, and wall slope are computed for the solution point and returned to subroutine DRWALL. If a tabular wall has been specified, then conical wall segments are fit between each two data points on the wall.

AXIS. Subroutine AXIS determines the solution at an axis point using the method of characteristics for rotational flow. The location and flow properties are known at points 1 and 3 in Fig. D-7. Since the flow is axisymmetric, the flow angle θ and y-coordinate at the axis must be zero. The compatibility equations along the right-running Mach line and the axis are solved to obtain the flow properties, p, ρ, and V. The equations programmed are as follows. Along the Mach line,

$$\frac{dy}{dx} = \tan(\theta - x) \qquad \text{(D-20)}$$

$$\frac{\sqrt{M^2 - 1}}{\rho V^2} dp + d\theta + \delta \left[\frac{v}{yMV \cos(\theta - \alpha)} \right] dx = 0 \qquad \text{(D-21)}$$

Along the axis,

$$\rho V dV + dp = 0 \qquad \text{(D-22)}$$

$$dp - a^2 d\rho = 0 \qquad \text{(D-23)}$$

The modified Euler predictor-corrector technique, based on empolying average properties for the corrector, is used in the numerical analysis. The remaining flow properties are obtained by calling subroutine THERMØ.

BASE. Subroutine BASE calculates the base pressure, the base thrust, and the total nozzle-base-boattail thrust. The exit lip point properties for the nozzle and the boattail are known from the nozzle and boattail flow field calculations. Using the known exit lip point properties in the base pressure model derived from the curve fit of Addy's (3) data, the base pressure and the base thrust are calculated. The total thrust is summed and printed out. Subroutine BASE also notes if the boattail exit radius is less than the nozzle exit radius, a physical impossibility. A message to this effect is printed out, and the program continues.

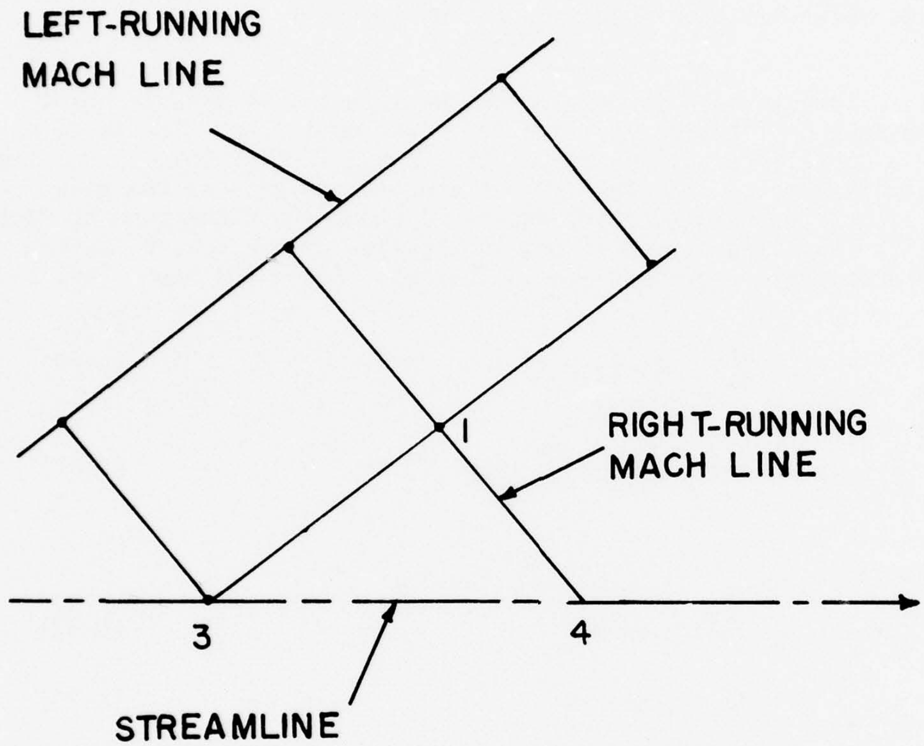


Figure D-7. Point Labeling Scheme for an Axis Point

TRAN2D. Subroutine TRAN2D transfers the properties stored at location D (I,J,N) in the D array to point 2.

TRAN1D. Subroutine TRAN1D transfers the properties stored at location D (I,J,N) in the D array to point 1.

TRAND4. Subroutine TRAND4 transfers the properties stored at point 4 to the D array at location D (I,J,N).

TRAN12. Subroutine TRAN12 transfers the properties stored at point 2 to point 1.

TRAN14. Subroutine TRAN14 transfers the properties stored at point 4 to point 1.

TRAN24. Subroutine TRAN24 transfers the properties stored at point 4 to point 2.

TRAN3D. Subroutine TRAN3D transfers the properties stored at location D (I,J,N) in the D array to point 3.

TRANSL. Subroutine TRANSL transfers the properties stored at point 4 to the secondary-start line array SL (K,JJP).

AXITER. Subroutine AXITER performs an axial iteration optimization of the objective function, thrust (PUSH). The subroutine requires an initial estimate (ANGA, RADB, ANGC, RADD) and increments (DA, DB, DC, DD) for each of the independent variables. The variables ANGC and DC are employed only when optimizing a boattail contour by itself. It then perturbs each of the variables in turn, fitting a quadratic polynomial of the form $y = a + bx + cx^2$ along the variable direction. The algorithm then finds the maximum point of the quadratic polynomial, calculates the objective function there, and steps to the maximum of the points on this line. The algorithm then proceeds to the next independent variable. Note that since all of the variables are independent, the search directions are orthogonal. If the algorithm encounters a prespecified boundary in any of the variables, this is noted and the variable is set equal to the boundary value. If the algorithm encounters an inequality constraint, such as the nozzle and boattail exit lip points coming closer than some previously specified value Δy_b, the variables become dependent ($y_{eb} = y_{en} + \Delta y_b$) and the variable space is diminished by one dimension.

The optimization may then proceed until convergence, when the function changes less than some specified relative tolerance. This algorithm stops in some (ITERØ) number of base point moves with an error message.

HILCLM. Subroutine HILCLM performs an optimization using the method of steepest descent (hill climbing). Beginning with an initial estimate (ANGA, RADB, ANGC, RADD) and increments (DA, DB, DC, DD), a first-order approximation to the unit gradient is calculated. The base point is then successively moved along the direction of the gradient until the objective function decreases. Then, the algorithm recalculates the gradient and begins again. This continues until the first step along a gradient no longer increases the objective function. Then the finite difference increments and step length are halved and the algorithm continues. Convergence is attained when the magnitude of the gradient becomes less than some specified relative tolerance. Some maximum number of gradient evaluations (ITERØ) stops the algorithm with an error message.

NEWTØN. Subroutine NEWTØN performs an optimization employing Newton's method. The algorithm requires a starting point (ANGA, RADB, ANGC, RADD) and increments (DA, DB, DC, DD) to form a matrix of second derivative approximations (Hessian) and a column vector along the negative gradient (see discussion in Section II). Subroutine GAUSS is employed to invert the Hessian and solve for the base point move, both in distance and direction. If this move exceeds the specified optimization boundaries (ANMAX, ANMIN), or the step does not increase the objective function, then some relaxation factor is employed which forces the step to remain in bounds. This then becomes the new base point, and the process repeats. Convergence is attained when the function changes less than some relative tolerance. If the algorithm does not converge within ITERØ base point moves, an error message is printed and the program stops.

GAUSS. Subroutine GAUSS is a simultaneous linear equation solver. A system of N linear equations, with coefficient matrix C and right hand side vector B is solved. The solution is the column vector V. The algorithm employs Gauss reduction, requiring only N - 1 passes to complete the reduction. Back substitution then fills the V vector before it returns to subroutine NEWTØN, the calling program.

APPENDIX E. INPUT PARAMETERS

The first data card in each run specifies the type of computation. The variable IØPT contained in column 1 selects whether an analysis (IØPT = 1) or an optimization (IØPT = 2) is performed. The variable ITYPE specified in column 2 determines the type of geometry being considered: ITYPE = 1 for a complete nozzle-base-boattail assembly, ITYPE = 2 for a nozzle only, and ITYPE = 3 for a boattail only. The remaining 78 spaces are available for any alphanumeric job title, which is printed at the beginning of each run.

All other input parameters are read in by subroutine INPUTT through a NAMELIST file called INFØ. Each of the parameters has a default value defined within the program. All of the input parameters necessary to perform nozzle, boattail, or nozzle-base-boattail analyses and optimizations are included in NAMELIST INFØ.

The code has the capability of using tabular values for gas properties, wall contours, and initial-value lines. If tabular gas properties are not input, the program uses a constant specific heat ratio and gas constant (table of 1 row). However, to use the tabular gas property option, the number of rows in the gas table NGASN or NGASB, respectively, (up to 20) must be specified. Each row includes a velocity magnitude, a specific heat ratio, and a gas constant.

Normally, the tabular wall contour option is not employed and the wall contour is internally computed as either a conical contour or a quadratic polynomial contour. When the tabular wall option is employed, the number of points on the tabular wall is given by NWALLN and NWALLB for the nozzle and boattail, respectively. Each point must have an x- and y-coordinate, up to a maximum of 100 wall points.

When the tabular initial-value line option is not utilized, an initial-value line is internally generated, using Kliegel's transonic analysis for the nozzle and a uniform flow for the boattail. The procedure for specifying tabular initial-value lines is discussed in the descriptions of subroutines IVLINN and IVLINB in Appendix D.

Tables E-1 through E-3 list the program input variables, a description of each variable, the engineering symbol for each variable, and the units and default value of each variable. When no units are specified, the variable is dimensionless. Figures E-1 and E-2 illustrate the nozzle and boattail geometries.

TABLE E.1. INPUT PARAMETERS FOR NOZZLE

Variable	Description	Units	Default Value
NPN	Number of points on the initial-value line.		11
IVLN	Type of initial-value line: 0 Kliegel, 1 read in points.		0
RATIØI	Geometric ratio of point spacing along the initial-value line.		5.0
IVLSN	Terminate analysis after nozzle initial-value line calculation; 0 no, 1 yes.		0
NPW	Number of points along throat initial expansion contour.		22
RATIØW	Geometric ratio of point spacing along throat expansion contour.		5.0
ICN	Number of applications of the corrector.		2
TØLN	Fractional convergence tolerance of the corrector.		0.001
KWRITN	Output option: 0 every point, 1 initial-value, axis, and wall points, 2 wall points, 3 exit lip point.		0
IDN	Dump flag: 0 no, 1 yes.		0
YTN	Throat radius, y_t.	in.	1.0
THETTN	Throat attachment angle, θ_{an}.	deg	30.0
RUN	Nozzle throat upstream radius of curvature, ρ_{tu}.	in.	3.0
RDN	Nozzle throat downstream radius of curvature, ρ_{td}.	in.	0.5
XEN	Nozzle exit length, x_{en}.	in.	10.0
YEN	Nozzle exit radius, y_{eb}.	in.	5.0
THETEN	Nozzle exit angle, θ_{an}.	deg	0.0
IWALLN	Wall contour: 1 cone, 2 second-order polynomial, 5 tabular.		2

TABLE E.1. CONTINUED

Variable	Description	Units	Default Value
DTHETN	Incremental angle change in Prandtl-Meyer expansion.	deg	1.0
DELTAN	1.0 axisymmetric nozzle, 0.0 planar nozzle.		1.0
PØN	Stagnation pressure, P_o.	psia	1000.0
TØN	Stagnation temperature, T_o.	R	6000.0
PAMB	Ambient pressure, p_{amb}.	psia	0.0
NGASN	Number of rows in gas property table, if 1, then constant γ and R.		1
VELN	Tabular values of velocity magnitude, V, NGASN values.	$\frac{ft}{sec}$	none
GAMN	Tabular values of specific heat ratio, γ, NGASN values.		1.2
RGASN	Tabular values of gas constant, R, NGASN values.	$\frac{ft\text{-}lbf}{lbm\text{-}R}$	60.0
NWALLN	Number of points on tabular wall.		none
XWN	x-coordinates of points on tabular wall, NWALLN values.	in.	none
YWN	y-coordinates of points on tabular wall, NWALLN values.	in.	none

TABLE E.2. INPUT PARAMETERS FOR BOATTAIL

Variable	Description	Units	Default Value
NPB	Number of points on the initial-value line.		30
IVLB	Type of initial-value line: 0 uniform, 1 read in points.		0
IVLSB	Terminate boattail analysis after boattail initial-value line calculation: 0 no, 1 yes.		0
ICB	Number of applications of the corrector.		2
TØLB	Fractional convergence tolerance of the corrector.		0.001
KWRITB	Output option: 0 every point, 1 initial-value, axis and wall points, 2 wall points, 3 exit lip point.		0
IDB	Dump flag: 0 no, 1 yes.		0
MS	Freestream Mach number.		1.4
SPACE	Spacing factor for points along the initial-value line.		0.5
YTB	Initial boattail radius, y_t.	in.	8.0
THETTB	Boattail attachment angle, θ_t.	deg	2.0
RDB	Boattail downstream radius of curvature, ρ_{td}.	in.	5.0
XEB	Boattail exit length, x_{eb}.	in.	10.0
YEB	Boattail exit radius, y_{eb}.	in.	7.0
THETEB	Boattail exit angle, θ_{eb}.	deg	0.0
IWALLB	Wall contour: 1 cone, 2 second-order polynomial, 3 cylindrical, 5 tabular		2
DTHETB	Incremental angle change in Prandtl-Meyer expansion.	deg	1.0

TABLE E.2. CONTINUED

Variable	Description	Units	Default Value
DELTAB	1.0 axisymmetric boattail, 0.0 planar boattail.		1.0
PØB	Stagnation pressure, P_0.	psia	12.0
TØB	Stagnation temperature, T_0.	R	600.0
NGASB	Number of rows in gas property table, if 1, then constant γ and R.		1
VELB	Tabular values of velocity magnitude, V, NGASB values.	$\frac{ft}{sec}$	none
GAMB	Tabular values of specific heat ratio γ, NGASB values.		1.4
RGASB	Tabular values of gas constant, R, NGASB values.	$\frac{ft\text{-}lbf}{lbm\text{-}R}$	53.3
NWALLB	Number of points on tabular wall.		none
XWB	x-coordinates of points on tabular wall, NWALLB values.	in.	none
YWB	y-coordinates of points on tabular wall, NWALLB values.	in.	none

TABLE E.3. INPUT PARAMETERS FOR OPTIMIZATION

Variable	Description	Units	Default Value
ANSTRT(1)	Initial estimate of nozzle throat attachment angle, θ_{an}.	deg	30.0
ANSTRT(2)	Initial estimate of the nozzle exit radius, y_{en}.	in.	5.0
ANSTRT(3)	Initial estimate of the boattail attachment angle, θ_{ab}. Only employed when designing a boattail alone.	deg	15.0
ANSTRT(4)	Initial estimate of the boattail exit radius, y_{eb}. Only employed when designing a boattail alone.	in.	4.0
ANMAX(1)	Maximum allowed nozzle throat attachment angle.	deg	45.0
ANMAX(2)	Maximum allowed nozzle exit radius.	in.	7.0
ANMAX(3)	Maximum allowed boattail attachment angle.	deg	20.0
ANMAX(4)	Maximum allowed boattail exit radius.	in.	8.0
ANMIN(1)	Minimum allowed nozzle throat attachment angle.	deg	20.0
ANMIN(2)	Minimum allowed nozzle exit radius.	in.	4.0
ANMIN(3)	Minimum allowed boattail attachment angle.	deg	0.0
ANMIN(4)	Minimum allowed boattail exit radius.	in.	4.0
IMETH	Optimization method: 1 axial iteration, 2 method of steepest descent, 3 Newton's method.		3
ITERØ	Maximum number of passes through the optimization algorithm.		10
TØLØ	Relative convergence tolerance for optimization		0.001

TABLE E.3. CONTINUED

Variable	Description	Units	Default Value
KWRITØ	Output option: 0 every point, 1 initial-value, axis, and wall points, 2 wall points, 3 exit lip point.		3
IØP	Specifies type of geometry for optimization: 1 nozzle, 2 boattail, 3 nozzle-base-boattail combination.		1
N	Specifies the dimension of the optimization space (i.e., the number of unconstrained parameters).		2
DA	Nozzle attachment angle perturbation.	deg	1.0
DB	Nozzle exit lip radius perturbation.	in.	$0.2y_{tn}$
DC	Boattail attachment angle perturbation.	deg	0.5
DD	Boattail exit lip radius perturbation.	in.	$0.2y_{tn}$
DBAS	Minimum annular base width, Δy_b.	in.	0.0

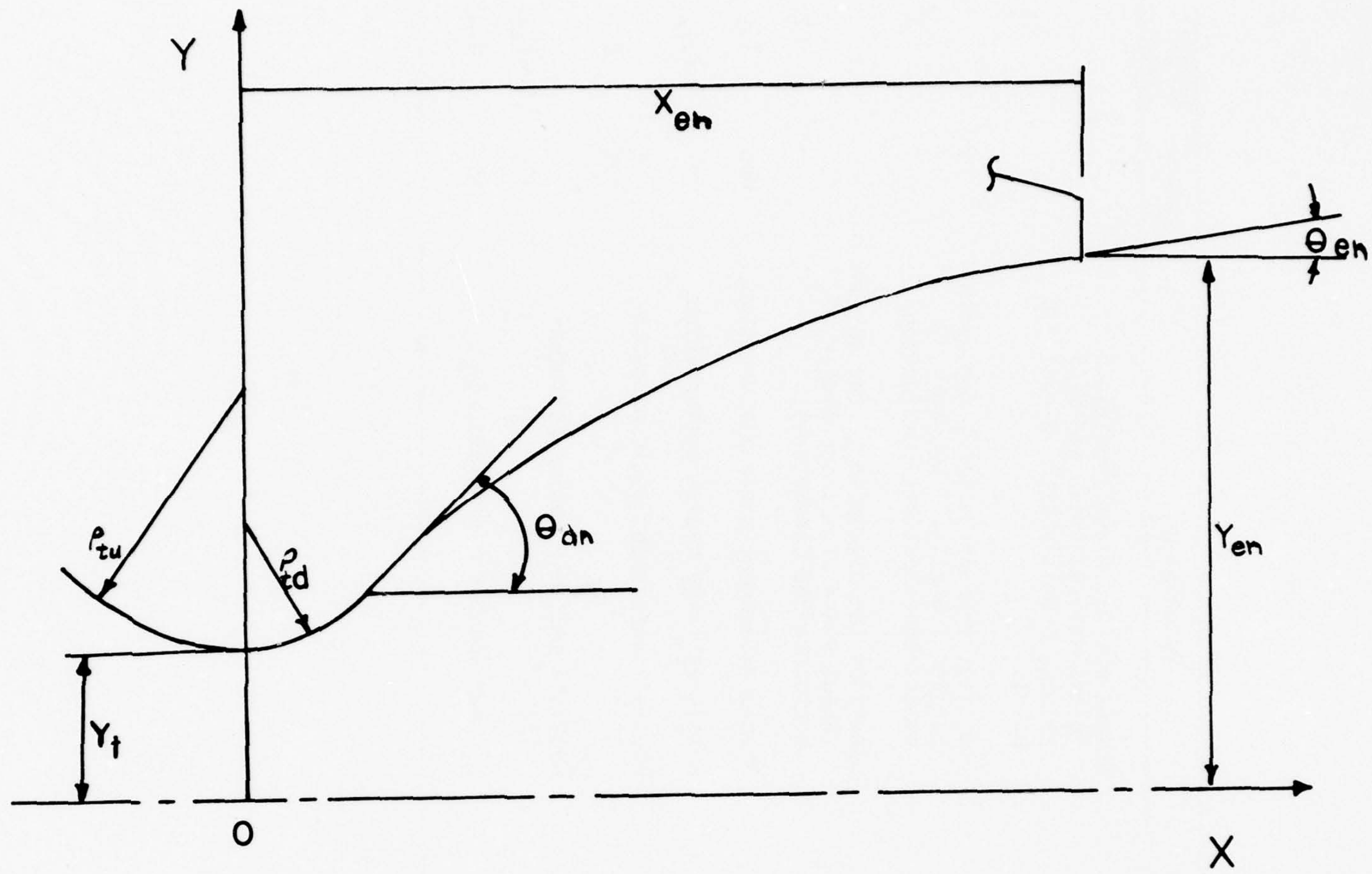


Figure E-1. Specification of the Nozzle Geometry

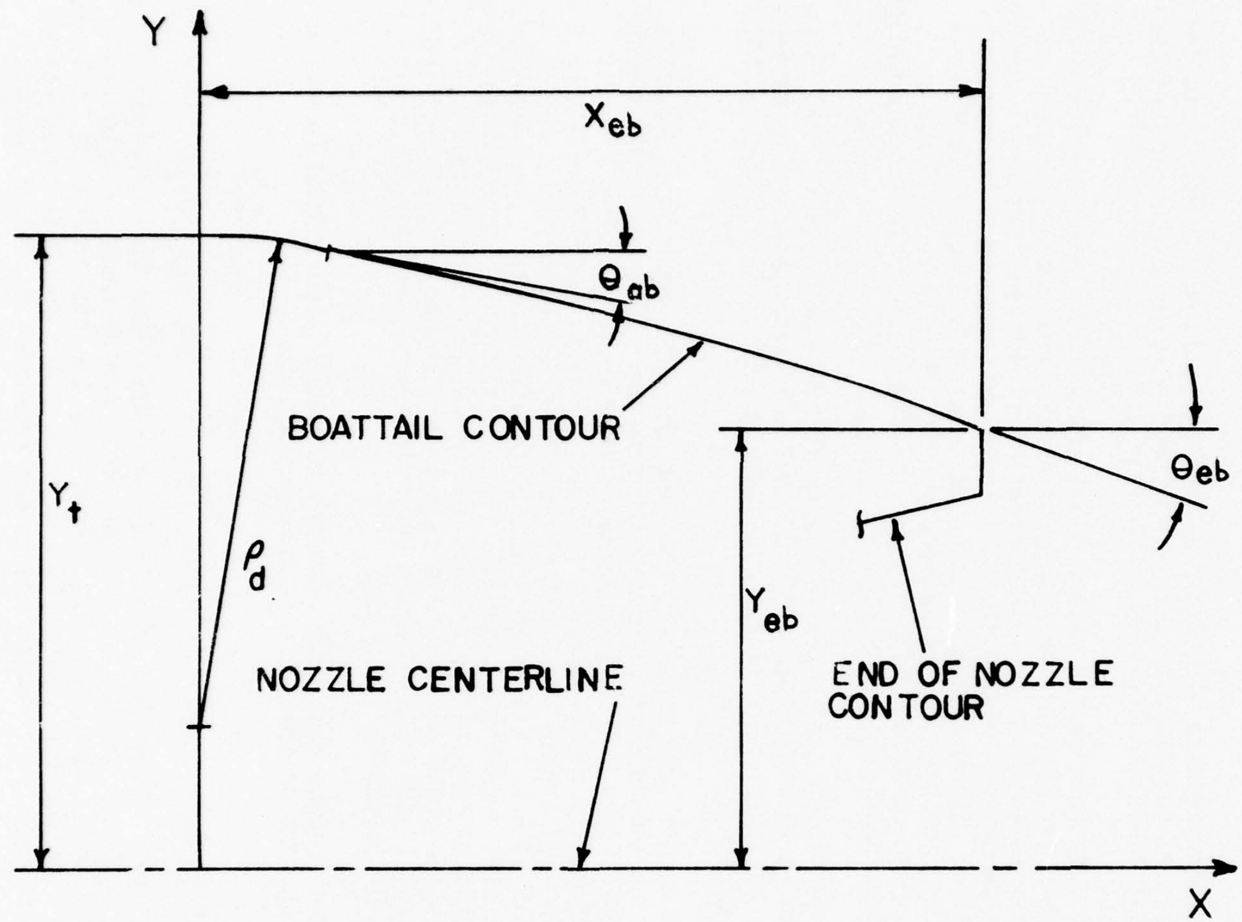


Figure E-2. Specification of the Boattail Geometry

APPENDIX F. SAMPLE CASES

Five sample cases are presented in this section to illustrate the application of the computer program. These cases exercise most of the options available for analysis or optimization.

Each data deck begins with a TITLE CARD. That card identifies the type of geometry (nozzle, boattail, or nozzle-base-boattail assembly) and whether an analysis or an optimization is desired. Column 1 contains the parameter IØPT which is 1 for an analysis and 2 for an optimization. Column 2 contains the parameter ITYPE, which is 1 for a complete nozzle-base-boattail assembly, 2 for a nozzle, or 3 for a boattail. The remainder of the card (78 columns) may contain any alphanumeric job identification data.

The following cards specify the remaining input data in free format form, read in through NAMELIST INFO. Not all variables need to be defined for each run, as default values are specified in subroutine INPUTT for most of the variables.

Each sample case considers the same thermodynamic model, which is specified by the default values in the program. The gas in the nozzle is a thermally and calorically perfect gas with γ = 1.2 and R = 60.0 (ft-lbf)/(lbm-R). Thus, NGASN = 1, GAMN(1) = 1.2, and RGASN(1) = 60.0. The gas flowing around the boattail is also a thermally and calorically perfect gas with γ = 1.4 and R = 53.3 (ft-lbf)/(lbm-R). Thus, NGASB = 1, GAMB(1) = 1.4, and RGASB(1) = 53.3. The nozzle gas properties are typical of high temperature exhaust jet gases, and the boattail gas properties are those of air. The stagnation properties are specified: in the nozzle, TØN = 6000.0 R and PØN = 1000 psia, and for the boattail TØB = 600.0 R and PØB = 12.0 psia.

In each sample case, the geometry to be analyzed is discussed, and the running time on the CDC 6500 computer is given.

1. Sample Case 1

The first sample case illustrates a nozzle flow field analysis. It considers a nozzle exhausting into a static region having a pressure of 5.0 psia (PAMB = 5.0). Thus, in column 1 of the title card IØPT = 1 and in column 2 of the title card ITYPE = 2. The job identification statement "SAMPLE CASE 1" is contained in the remainder of the title card.

A Kliegel type initial-value line is specified by default (IVLN = 0). The nozzle geometry is specified with the nozzle upstream radius of curvature RUN = 3.0 in., the nozzle downstream radius of curvature RDN = 0.5 in., the nozzle throat radius YTN = 1.0 in., the throat attachment angle THETTN = 30.0 deg, the nozzle exit radius YEN = 5.0 in., and the nozzle length XEN = 10.0 in. The nozzle wall is a second-order polynomial contour (IWALLN = 2). All of the parameters are the program default values.

The accuracy of the analysis depends on the number and spacing of the initial-value line points. In this sample case, 11 points are specified spanning the nozzle throat radius (NPN = 11).

The best accuracy is obtained when these points are spaced geometrically across the throat with the ratio of the distance between the two points at the centerline to the distance between the two points at the wall having a value larger than unity. For this sample case, RATIØI = 5.0. That spacing makes the change in property values between points of the computational mesh roughly comparable, because property gradients are greatest near the wall. A similar procedure is employed for points along the nozzle wall on the initial expansion contour. A wall point spacing compatible with the initial-value line spacing is obtained with NPW = 22 and RATIØW = 5.0. It should be noted that the number of points in the flow field solution mesh, and hence the computation time, is proportional to the square of NPN. Large amounts of computation time may be saved by reducing NPN, although a corresponding decrease in accuracy results. The above parameters are all specified by the program default values.

The parameters TØLN and ICN are the fractional convergence tolerance and the maximum number of applications of the corrector, respectively, in the method of characteristics unit processes. Those parameters are specified by their default values (TØLN = 0.001 and ICN = 2).

Output is kept brief by setting KWRITN = 1. This option prints only initial-value line points, axis points, wall points, and the one-dimensional approximation to the solution at the wall points.

The following list presents the data deck for sample case 1. Note that only KWRITN and PAMB are specified, because default values are employed for all of the other parameters.

```
12   SAMPLE CASE 1   NOZZLE ANALYSIS
 $INFO     KWRITN = 1, PAMB = 5.0 $
```

A complete listing of the output from sample case 1 is presented in Fig. F-1. The first page contains the program abstract and the job title. The nozzle flow field analysis data and initial-value line are listed on the second page. Column headings for the initial-value line data are as follows.

X	downstream distance from the nozzle throat, in.
Y	radial distance from the nozzle axis, in.
M	Mach number
P	pressure, psia
Q	velocity magnitude, ft/sec
TH	flow angle, deg.
U	x-component of velocity, ft/sec
V	y-component of velocity, ft/sec
R	density, lbm/ft^3
T	temperature, R
WDØT	mass flow rate between the axis of symmetry and the given point, lbm/sec
THRUST	thrust between the axis of symmetry and the given point, lbf
ISP	specific impulse for the flow between the axis of symmetry and the given point, lbf-sec/lbm

The initial-value line performance parameters are also listed.

The next three pages of output presented in Fig. F-1 contain the results of the method of characteristics flow field analysis. The first of these three pages presents the wall contour specification and the column headings for the method of characteristics calculations. J and I are the characteristic coordinates, and the next ten columns are the same as described above for the initial-value line output. Columns 13 and 14, headed by MDT/LRC and F/LRC, respectively, refer to the integrated values of the mass flow rate and the thrust along the left-running Mach line from the axis of symmetry to the given point. These two columns represent checks on the accuracy of the program. The mass flow rate at any wall point should be close to the mass flow rate at the throat. The thrust obtained along the wall, THRUST, should be close to the value obtained along the left-running characteristic, F/LRC. The specific impulse, ISP, is printed in the next column, and the number of applications of the corrector, IC, or the specific impulse efficiency, EFF, is printed in the last column. IC is printed when the point is a flow field point computed by any of the method of characteristics unit process, and EFF is printed when the point is the one-dimensional approximation to the wall point solution. When 0 is printed in the last column, the point is on the initial-value line. A -2 in the last column indicates that the point is the nozzle exit lip.

Wall points are located at the end of every left-running Mach line. However, in the throat region of the nozzle, the inverse wall point method is employed. That procedure arbitrarily specifies the wall point locations, thus controlling the spacing and enhancing the accuracy of the flow field analysis. In this region there may be several wall points along the same left-running characteristic. After a wall point is printed, the one-dimensional flow approximation to the solution at the wall point is computed and printed for reference.

The last line of the output gives the total number of characteristics that crossed inside of the nozzle. The formation of oblique shock waves is indicated when this is nonzero.

Sample case 1 required 5 seconds of central processor time on the CDC 6500 computer.

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          ANALYSIS AND DESIGN OF SUPERSONIC NOZZLE-BASE-BOATTAIL COMBINATIONS

ABSTRACT

     THIS PROGRAM WAS PRODUCED AT THE PURDUE UNIVERSITY THERMAL SCIENCE AND PROPULSION CENTER BY J. G. ALLMAN AS A PART OF THE
     REQUIREMENTS OF AF CONTRACT NUMBER F33615-73-C-2010.  THE CONTRACT WAS SPONSORED BY THE AERO PROPULSION LABORATORY WRIGHT
     PATTERSON AFB, OHIO.  PRINCIPAL INVESTIGATOR FOR PURDUE UNIVERSITY WAS PROFESSOR JOE D. HOFFMAN.

     THE EQUATIONS OF MOTION FOR AN AXISYMMETRIC SUPERSONIC FLOW ARE SOLVED USING A NUMERICAL METHOD OF CHARACTERISTICS HAVING
     SECOND-ORDER ACCURACY.  THE INITIAL FLOW VARIABLES FOR THE NOZZLE FLOW FIELD CAN BE SPECIFIED OR COMPUTED FROM KLIEGELS
     TRANSONIC FLOW ANALYSIS.  THE INITIAL FLOW VARIABLES FOR THE BOATTAIL FLOW CAN BE SPECIFIED OR COMPUTED FROM A UNIFORM
     FLOW ANALYSIS.  THE NOZZLE AND BOATTAIL GEOMETRIES CAN BE SPECIFIED AS CONICAL, SECOND-ORDER POLYNOMIAL OR TABULAR CONTOUR
     THE ANALYSIS WAS BASED ON THE GOVERNING GAS DYNAMIC RELATIONS FOR A ROTATIONAL FLOW OF A FROZEN OR EQUILIBRIUM GAS MIXTURE


JOB TITLE

     SAMPLE CASE 1      NOZZLE ANALYSIS
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Figure F-1. Output for Sample Case 1.

NOZZLE FLOW FIELD ANALYSIS

NP = 11	TO = 6000.00 R	XE =10.000 IN	G = 1.20
IC = 2	PO = 1000.00 PSIA	YE = 5.000 IN	RG = 60.00 (FT-LBF)/(LBM-R)
IVL = 0	PAMB = 5.00 PSIA	RU = 3.000 IN	NGAS = 1
IWALL = 2		RD = .500 IN	RATIOI =5.00
NPW = 22		YT = 1.000 IN	RATIOW =5.00
KWRITE =1			TOL = .0010

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A KLIEGEL V=0 LINE

I	X	Y	M	P	Q	TH	U	V	R	T	WDOT	THRUST	ISP
1	0.000	1.000	1.082	514.66	3816.6	-.000	3816.6	-.0	2.2997E-01	5371.2	19.221	3881.113	201.916
2	.012	.961	1.075	519.00	3793.7	.004	3793.7	.3	2.3158E-01	5378.7	17.744	3582.427	201.892
3	.025	.914	1.067	523.76	3768.6	.010	3768.6	.7	2.3335E-01	5386.9	16.054	3240.797	201.867
4	.040	.857	1.058	528.90	3741.6	.019	3741.6	1.3	2.3525E-01	5395.7	14.143	2854.574	201.843
5	.057	.790	1.049	534.33	3713.0	.031	3713.0	2.0	2.3726E-01	5404.9	12.014	2424.716	201.820
6	.075	.710	1.040	539.92	3683.6	.043	3683.6	2.8	2.3933E-01	5414.3	9.696	1956.556	201.799
7	.094	.614	1.031	545.49	3654.3	.054	3654.3	3.4	2.4139E-01	5423.5	7.248	1462.456	201.781
8	.114	.499	1.022	550.76	3626.6	.059	3626.6	3.7	2.4333E-01	5432.2	4.786	965.746	201.765
9	.132	.361	1.015	555.39	3602.4	.054	3602.4	3.4	2.4503E-01	5439.8	2.511	506.534	201.753
10	.146	.197	1.009	558.82	3584.3	.034	3584.3	2.1	2.4629E-01	5445.4	.745	150.233	201.746
11	.151	0.000	1.007	560.23	3577.0	0.000	3577.0	0.0	2.4681E-01	5447.7	0.000	0.000	0.000

INITIAL-VALUE LINE PERFORMANCE PARAMETERS

THE NOZZLE MASS FLOW RATE IS 19.221 (LBM/SEC)

THE START LINE THRUST IS 3881.1 LBF, AND THE START LINE ISP IS 201.916 (LBF-SEC)/LBM

THE ONE-DIMENSIONAL MASS FLOW RATE IS 19.261 (LBM/SEC), AND THE DISCHARGE COEFFICIENT IS .9979

THE ONE-DIMENSIONAL THRUST IS 3896.9 LBF, AND THE ONE-DIMENSIONAL SPECIFIC IMPULSE IS 202.318 (LBF-SEC/LBM)

THE SPECIFIC IMPULSE EFFICIENCY IS .9980

Figure F-1. Output for Sample Case 1 (Continued).

THE SUPERSONIC WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT = 30.000 DEGREES AND YE = 5.000 IN

J	I	X	Y	M	P	Q	TH	U	V	R	T	MOT/LRC	F/LRC	THRUST	ISP	IC/EF
1	11	0.000	1.000	1.082	514.663	3816.6	-.000	3816.6	-.0	2.2997E-01	5371.2	19.221	3881.1			0
2	10	.012	.961	1.075	518.998	3793.7	.004	3793.7	.3	2.3158E-01	5378.7	17.744	3582.4			0
1	12	.005	1.000	1.107	499.791	3895.2	.543	3895.0	36.9	2.2441E-01	5345.0	17.744	3582.4	3865.5	201.102	2
		.005	1.000	1.007	560.141	3577.4				2.4678E-01	5447.5	19.261		3885.7	201.740	.9968
1	13	.010	1.000	1.132	485.317	3972.1	1.130	3971.3	78.3	2.1899E-01	5318.9	17.946	3623.4	3865.7	201.114	2
		.010	1.000	1.015	555.457	3602.0				2.4506E-01	5439.9	19.261		3886.0	201.753	.9968
1	14	.015	1.000	1.156	470.917	4049.0	1.763	4047.1	124.6	2.1356E-01	5292.3	18.242	3683.5	3866.1	201.136	2
		.015	1.000	1.023	550.395	3628.6				2.4320E-01	5431.6	19.261		3886.5	201.778	.9968
1	15	.021	1.000	1.182	456.380	4127.2	2.447	4123.4	176.2	2.0805E-01	5264.7	18.491	3734.4	3866.8	201.169	2
		.021	1.000	1.032	544.926	3657.3				2.4118E-01	5422.6	19.261		3887.2	201.817	.9968
1	16	.028	1.001	1.208	441.520	4207.7	3.185	4201.2	233.8	2.0239E-01	5235.7	18.713	3780.0	3867.6	201.215	2
		.028	1.001	1.042	539.017	3686.3				2.3900E-01	5412.8	19.261		3888.3	201.872	.9967
3	9	.025	.914	1.067	523.758	3768.6	.010	3768.6	.7	2.3335E-01	5386.9	16.054	3240.8			0
2	17	.035	1.001	1.236	426.189	4291.5	3.982	4281.1	298.0	1.9652E-01	5204.9	17.796	3596.4	3868.8	201.276	2
		.035	1.001	1.052	532.634	3721.9				2.3664E-01	5402.0	19.261		3889.7	201.948	.9967
2	18	.042	1.002	1.284	410.783	4376.6	4.843	4361.0	369.5	1.9058E-01	5173.1	18.040	3646.9	3870.3	201.354	2
		.042	1.002	1.064	525.744	3758.1				2.3408E-01	5390.3	19.261		3891.6	202.046	.9966
2	19	.050	1.003	1.294	394.877	4465.5	5.772	4442.8	449.1	1.8441E-01	5139.2	18.269	3694.9	3872.2	201.451	2
		.050	1.003	1.076	518.308	3797.3				2.3132E-01	5377.5	19.261		3894.1	202.173	.9964
2	20	.059	1.003	1.326	378.388	4558.9	6.775	4527.1	537.8	1.7797E-01	5102.7	18.468	3740.8	3874.5	201.571	2
		.059	1.003	1.089	510.288	3839.7				2.2834E-01	5363.6	19.261		3897.2	202.333	.9962
2	21	.068	1.005	1.360	361.185	4658.0	7.858	4614.3	636.8	1.7120E-01	5063.3	18.698	3785.2	3877.3	201.715	2
		.068	1.005	1.104	501.644	3885.4				2.2511E-01	5348.3	19.261		3901.0	202.530	.9960
4	8	.040	.857	1.058	528.895	3741.6	.019	3741.6	1.3	2.3525E-01	5395.7	14.143	2854.6			0
3	22	.076	1.006	1.397	343.203	4763.5	9.027	4704.5	747.4	1.6407E-01	5020.4	17.862	3625.9	3880.5	201.886	2
		.076	1.006	1.120	492.335	3934.8				2.2162E-01	5331.6	19.261		3905.6	202.772	.9956
3	23	.089	1.008	1.434	325.382	4870.3	10.289	4792.0	869.9	1.5694E-01	4975.9	18.118	3676.7	3884.4	202.086	2
		.089	1.008	1.137	482.321	3988.1				2.1786E-01	5313.4	19.261		3911.3	203.064	.9952
3	24	.101	1.010	1.475	306.916	4983.6	11.652	4880.9	1006.5	1.4948E-01	4927.7	18.345	3726.2	3888.9	202.319	2
		.101	1.010	1.155	471.561	4045.6				2.1380E-01	5293.5	19.261		3918.0	203.416	.9946
3	25	.114	1.013	1.519	287.758	5104.5	13.123	4971.2	1159.0	1.4167E-01	4875.0	18.564	3774.2	3894.0	202.585	2
		.114	1.013	1.175	460.016	4107.6				2.0943E-01	5271.7	19.261		3926.1	203.835	.9939
3	26	.127	1.016	1.567	267.788	5234.6	14.712	5063.0	1329.4	1.3342E-01	4816.9	18.774	3820.9	3899.8	202.887	2
		.127	1.016	1.197	447.651	4174.4				2.0473E-01	5247.8	19.261		3935.6	204.330	.9929
5	7	.057	.790	1.049	534.326	3713.0	.031	3713.0	2.0	2.3726E-01	5404.9	12.014	2424.7			0
4	27	.141	1.020	1.619	247.492	5371.9	16.427	5152.6	1519.2	1.2494E-01	4754.0	18.058	3688.5	3906.3	203.225	2
		.141	1.020	1.221	434.435	4246.3				1.9968E-01	5221.6	19.261		3946.8	204.912	.9918
4	28	.157	1.025	1.673	227.484	5513.0	18.279	5234.8	1729.1	1.1647E-01	4687.6	18.293	3741.7	3913.5	203.599	2
		.157	1.025	1.247	420.345	4323.7				1.9427E-01	5193.0	19.261		3959.9	205.591	.9903
4	29	.173	1.031	1.732	207.096	5663.8	20.278	5312.7	1963.0	1.0771E-01	4614.7	18.517	3793.1	3921.4	204.010	2
		.173	1.031	1.274	405.367	4406.8				1.8848E-01	5161.7	19.261		3975.1	206.378	.9885
4	30	.191	1.038	1.797	186.404	5825.5	22.437	5384.5	2223.4	9.6663E-02	4534.3	18.729	3842.3	3929.9	204.455	2
		.191	1.038	1.304	389.496	4495.9				1.8231E-01	5127.5	19.261		3992.6	207.286	.9863
6	6	.075	.710	1.040	539.919	3683.6	.043	3683.6	2.8	2.3933E-01	5414.3	9.696	1956.6			0
5	31	.209	1.046	1.869	165.641	5998.5	24.767	5446.7	2513.0	8.9420E-02	4445.8	18.097	3735.3	3939.0	204.929	2
		.209	1.046	1.337	372.748	4591.3				1.7575E-01	5090.0	19.261		4012.6	208.327	.9837

Figure F-1. Output for Sample Case 1 (Continued).

J	I	X	Y	M	P	Q	TH	U	V	R	T	MDT/LRC	F/LRC	THRUST	ISP	IC/EFF
5	32	.229	1.056	1.946	145.657	6177.7	27.283	5490.4	2831.8	8.0339E-02	4351.3	18.329	3790.3	3948.6	205.428	2
		.229	1.056	1.372	355.153	4693.2				1.6881E-01	5049.2	19.261		4035.4	209.511	.9805
5	33	.250	1.067	2.030	126.088	6368.3	30.000	5515.1	3184.2	7.1244E-02	4247.5	18.546	3842.1	3958.5	205.944	2
		.250	1.067	1.410	336.766	4801.9				1.6150E-01	5004.6	19.261		4061.2	210.852	.9767
6	34	.294	1.092	2.046	122.646	6403.8	29.933	5549.6	3195.4	6.9619E-02	4228.0	19.225	4006.1	3979.0	207.010	1
		.294	1.092	1.485	302.404	5011.9				1.4764E-01	4915.7	19.261		4115.1	213.649	.9689
7	5	.094	.614	1.031	545.486	3654.3	.054	3654.3	3.4	2.4139E-01	5423.5	7.248	1462.5			0
7	35	.400	1.153	2.082	115.203	6482.7	29.769	5627.2	3218.7	6.6081E-02	4184.1	19.226	4059.5	4028.0	209.557	2
		.400	1.153	1.632	242.306	5408.1				1.2275E-01	4737.5	19.261		4229.0	219.563	.9544
8	4	.114	.499	1.022	550.763	3626.6	.059	3626.6	3.7	2.4333E-01	5432.2	4.786	965.7			0
8	36	.521	1.222	2.123	107.223	6571.0	29.583	5714.4	3244.0	6.2244E-02	4134.3	19.227	4118.9	4082.6	212.397	2
		.521	1.222	1.767	195.813	5751.1				1.0278E-01	4572.2	19.261		4338.2	225.233	.9430
9	3	.132	.361	1.015	555.385	3602.4	.054	3602.4	3.4	2.4503E-01	5439.8	2.511	506.5			0
9	37	.657	1.299	2.169	98.854	6668.3	29.372	5811.1	3270.6	5.8169E-02	4078.6	19.228	4183.7	4142.3	215.503	2
		.657	1.299	1.894	158.915	6057.9				8.6370E-02	4415.8	19.261		4442.5	230.645	.9344
10	2	.146	.197	1.009	558.817	3584.3	.034	3584.3	2.1	2.4629E-01	5445.4	.745	150.2			0
10	38	.807	1.383	2.220	90.367	6772.6	29.138	5915.5	3297.7	5.3977E-02	4018.0	19.228	4252.4	4205.8	218.806	2
		.807	1.383	2.014	129.597	6333.7				7.2871E-02	4268.3	19.261		4540.5	235.733	.9282
11	1	.151	0.000	1.007	560.226	3577.0	0.000	3577.0	0.0	2.4681E-01	5447.7	0.000	0.0			0
11	39	.969	1.473	2.274	82.064	6881.2	28.885	6025.1	3324.0	4.9813E-02	3953.9	19.228	4322.7	4271.2	222.210	2
		.969	1.473	2.128	106.429	6581.0				6.1841E-02	4130.4	19.261		4631.0	240.432	.9242
12	2	.178	0.000	1.019	552.901	3615.4	0.000	3615.4	0.0	2.4412E-01	5435.7	0.000	0.0			1
12	40	1.113	1.552	2.321	75.358	6974.4	28.658	6120.0	3344.8	4.6397E-02	3898.0	19.229	4382.3	4326.7	225.100	2
		1.113	1.552	2.218	90.723	6769.2				5.4137E-02	4022.0	19.261		4701.2	244.078	.9222
13	3	.204	0.000	1.033	544.326	3660.4	0.000	3660.4	0.0	2.4096E-01	5421.6	0.000	0.0			1
13	41	1.237	1.619	2.361	70.115	7051.3	28.464	6198.9	3360.7	4.3692E-02	3851.4	19.229	4431.3	4372.1	227.458	2
		1.237	1.619	2.288	79.934	6911.4				4.8716E-02	3938.0	19.261		4754.8	246.861	.9214
14	4	.230	0.000	1.047	535.922	3704.6	0.000	3704.6	0.0	2.3785E-01	5407.6	0.000	0.0			1
14	42	1.351	1.681	2.397	65.639	7120.2	28.283	6270.1	3373.7	4.1354E-02	3809.4	19.229	4475.1	4412.3	229.549	1
		1.351	1.681	2.349	71.595	7030.5				4.4442E-02	3866.3	19.261		4800.0	249.205	.9211
15	5	.257	0.000	1.061	527.215	3750.4	0.000	3750.4	0.0	2.3463E-01	5392.8	0.000	0.0			1
15	43	1.463	1.741	2.432	61.585	7185.3	28.106	6338.0	3385.0	3.9215E-02	3769.1	19.230	4516.4	4450.0	231.512	1
		1.463	1.741	2.405	64.647	7137.1				4.0818E-02	3801.1	19.261		4840.5	251.312	.9212
16	6	.284	0.000	1.076	518.325	3797.2	0.000	3797.2	0.0	2.3133E-01	5377.6	0.000	0.0			1
16	44	1.574	1.800	2.466	57.641	7248.2	27.928	6404.1	3394.8	3.7217E-02	3729.9	19.230	4556.2	4486.1	233.389	1
		1.574	1.800	2.458	58.686	7235.1				3.7656E-02	3740.3	19.261		4877.9	253.249	.9216
17	7	.312	0.000	1.091	509.325	3844.8	0.000	3844.8	0.0	2.2798E-01	5361.9	0.000	0.0			1
17	45	1.685	1.859	2.499	54.370	7309.2	27.750	6468.5	3403.2	3.5346E-02	3691.7	19.230	4594.6	4520.7	235.188	1
		1.685	1.859	2.508	53.518	7325.8				3.4872E-02	3683.3	19.261		4912.4	255.042	.9222
18	8	.340	0.000	1.106	500.436	3891.8	0.000	3891.8	0.0	2.2465E-01	5346.2	0.000	0.0			1

Figure F-1. Output for Sample Case 1 (Continued).

J I	X	Y	M	P	G	TH	U	V	R	T	MDT/LRC	F/LRC	THRUST	ISP	TC/EFF
18 46	1.795	1.917	2.532	51.207	7367.1	27.574	6530.3	3410.2	3.3624E-02	3655.0	19.231	4631.0	4553.3	236.885	1
	1.795	1.917	2.555	49.075	7408.9				3.2442E-02	3630.5	19.261		4943.9	256.680	.9229
19 9	.366	0.000	1.120	491.858	3937.3	0.000	3937.3	0.0	2.2144E-01	5330.8	0.000	0.0			1
19 47	1.902	1.972	2.563	48.348	7421.6	27.402	6580.9	3415.7	3.2053E-02	3620.2	19.231	4665.2	4583.7	238.467	1
	1.902	1.972	2.598	45.266	7484.4				3.0330E-02	3581.9	19.261		4972.6	258.165	.9237
20 10	.392	0.000	1.134	483.769	3980.4	0.000	3980.4	0.0	2.1840E-01	5316.1	0.000	0.0			1
20 48	2.005	2.025	2.592	45.796	7472.3	27.237	6643.8	3419.8	3.0636E-02	3587.6	19.231	4696.9	4611.7	239.923	1
	2.005	2.025	2.638	42.018	7552.4				2.8505E-02	3537.7	19.261		4998.2	259.498	.9246
21 11	.415	0.000	1.147	476.293	4020.3	0.000	4020.3	0.0	2.1559E-01	5302.3	0.000	0.0			1
21 49	2.104	2.076	2.620	43.447	7520.7	27.076	6696.5	3423.2	2.9321E-02	3556.3	19.231	4726.6	4637.7	241.276	1
	2.104	2.076	2.675	39.197	7614.7				2.6901E-02	3497.0	19.261		5021.6	260.711	.9255
22 12	.545	0.000	1.245	421.426	4317.5	0.000	4317.5	0.0	1.9469E-01	5195.0	0.000	0.0			2
22 50	2.658	2.354	2.767	33.064	7759.8	26.171	6964.3	3422.5	2.3365E-02	3396.2	19.232	4874.3	4766.1	247.959	2
	2.658	2.354	2.860	27.719	7907.5				2.0155E-02	3300.8	19.261		5129.3	266.300	.9311
23 13	.656	0.000	1.332	375.125	4577.3	0.000	4577.3	0.0	1.7670E-01	5095.0	0.000	0.0			2
23 51	3.162	2.597	2.885	26.457	7942.1	25.334	7178.3	3398.4	1.9409E-02	3271.5	19.232	4986.9	4859.9	252.838	2
	3.162	2.597	3.001	21.215	8115.5				1.6128E-02	3156.9	19.261		5202.0	270.077	.9362
24 14	.761	0.000	1.420	332.285	4828.2	0.000	4828.2	0.0	1.5972E-01	4992.9	0.000	0.0			2
24 52	3.674	2.835	2.992	21.631	8097.8	24.474	7370.2	3354.7	1.6414E-02	3162.8	19.231	5084.1	4937.1	256.853	2
	3.674	2.835	3.125	16.771	8286.8				1.3259E-02	3035.6	19.261		5258.1	272.989	.9409
25 15	.867	0.000	1.505	293.605	5066.6	0.000	5066.6	0.0	1.4408E-01	4890.7	0.000	0.0			2
25 53	4.208	3.073	3.090	17.972	8234.0	23.562	7547.5	3291.5	1.4067E-02	3066.2	19.230	5171.4	5002.5	260.255	2
	4.208	3.073	3.238	13.530	8434.7				1.1087E-02	2928.9	19.261		5302.5	275.293	.9454
26 16	.976	0.000	1.593	257.266	5304.4	0.000	5304.4	0.0	1.2906E-01	4783.9	0.000	0.0			2
26 54	4.789	3.320	3.183	15.067	8357.8	22.557	7718.4	3206.0	1.2147E-02	2977.0	19.228	5253.2	5059.7	263.233	2
	4.789	3.320	3.346	11.025	8568.5				9.3476E-03	2830.6	19.261		5338.2	277.150	.9498
27 17	1.091	0.000	1.686	223.023	5544.4	0.000	5544.4	0.0	1.1459E-01	4671.1	0.000	0.0			2
27 55	5.433	3.580	3.271	12.734	8471.0	21.424	7885.6	3094.2	1.0559E-02	2894.3	19.226	5331.2	5109.9	265.844	2
	5.433	3.580	3.451	9.043	8692.0				7.9245E-03	2738.6	19.261		5366.3	278.607	.9542
28 18	1.214	0.000	1.782	191.257	5785.7	0.000	5785.7	0.0	1.0083E-01	4552.6	0.000	0.0			2
28 56	6.149	3.852	3.355	10.870	8573.2	20.145	8048.8	2952.6	9.2552E-03	2818.7	19.224	5405.6	5153.1	268.089	2
	6.149	3.852	3.552	7.469	8805.7				6.7574E-03	2652.7	19.261		5386.7	279.665	.9586
29 19	1.350	0.000	1.884	161.715	6031.4	0.000	6031.4	0.0	8.7680E-02	4426.5	0.000	0.0			2
29 57	6.959	4.137	3.433	9.377	8665.3	18.673	8209.2	2774.3	8.1836E-03	2750.0	19.220	5477.5	5189.7	269.998	2
	6.959	4.137	3.651	6.202	8911.5				5.7880E-03	2571.9	19.261		5399.6	280.334	.9631
30 20	1.501	0.000	1.993	134.388	6284.0	0.000	6284.0	0.0	7.5158E-02	4291.4	0.000	0.0			2
30 58	7.883	4.434	3.505	8.196	8746.4	16.960	8366.0	2551.3	7.3153E-03	2688.8	19.216	5547.1	5220.0	271.574	2
	7.883	4.434	3.747	5.183	9009.5				4.9833E-03	2496.0	19.261		5404.9	280.610	.9678
31 21	1.672	0.000	2.112	109.417	6544.5	0.000	6544.5	0.0	6.3339E-02	4146.0	0.000	0.0			2
31 59	8.945	4.738	3.567	7.287	8815.1	14.953	8516.6	2274.6	6.6334E-03	2636.5	19.211	5613.8	5244.0	272.822	2
	8.945	4.738	3.838	4.369	9099.1				4.3220E-03	2425.9	19.261		5402.7	280.499	.9726
32 22	1.871	0.000	2.242	86.929	6814.2	0.000	6814.2	0.0	5.2304E-02	3988.7	0.000	0.0			2
32 60	10.000	5.000	3.609	6.727	8860.9	12.[illegible]	8636.5	1981.4	6.2058E-03	2601.6	19.206	5669.2	5260.1	273.659	-2
	10.000	5.000	3.913	3.804	9169.3				3.8514E-03	2370.6	19.261		5395.3	280.114	.9770

0 CHARACTERISTICS CROSSED IN THE NOZZLE.

Figure F-1. Output for Sample Case 1 (Concluded).

2. Sample Case 2

Sample case 2 illustrates a boattail flow field analysis. Thus, the title cards contains 13 in the first two columns. A uniform flow initial-value line is specified by default (IVLB = 0). The wall geometry is specified with outer radius YTB = 8.0 in., initial expansion contour radius RDB = 5.0 in., attachment angle THETTB = 2.0 deg, exit radius YEB = 7.0 in., and length XEB = 10.0 in. The boattail wall is the second-order polynomial (IWALLB = 2). All of these parameters are the default values given in the program. For this sample case, KWRITB = 2 for abbreviated output. This option dictates that only wall points are printed out. The thrust values printed in the last column are obtained by integrating the pressure forces acting on the boattail contour. The following list presents the data deck for sample case 2.

```
13   SAMPLE CASE 2    BOATTAIL ANALYSIS
 $INFO  KWRITB=2 $
```

Figure F-2 presents the output for sample case 2. The first page presents the program abstract and the job title. The second page presents boattail flow field analysis data and the initial-value line. The boattail geometric specification is given at the top of the third page. The third and fourth pages present the boattail flow field solution.

Sample case 2 required 4 seconds of central processor time on the CDC 6500 computer.

```
            ANALYSIS AND DESIGN OF SUPERSONIC NOZZLE-BASE-BOATTAIL COMBINATIONS

ABSTRACT

     THIS PROGRAM WAS PRODUCED AT THE PURDUE UNIVERSITY THERMAL SCIENCE AND PROPULSION CENTER BY J. G. ALLMAN AS A PART OF THE
     REQUIREMENTS OF AF CONTRACT NUMBER F33615-73-C-2010.  THE CONTRACT WAS SPONSORED BY THE AERO PROPULSION LABORATORY WRIGHT
     PATTERSON AFB, OHIO.  PRINCIPAL INVESTIGATOR FOR PURDUE UNIVERSITY WAS PROFESSOR JOE D. HOFFMAN.

     THE EQUATIONS OF MOTION FOR AN AXISYMMETRIC SUPERSONIC FLOW ARE SOLVED USING A NUMERICAL METHOD OF CHARACTERISTICS HAVING
     SECOND-ORDER ACCURACY.  THE INITIAL FLOW VARIABLES FOR THE NOZZLE FLOW FIELD CAN BE SPECIFIED OR COMPUTED FROM KLIEGELS
     TRANSONIC FLOW ANALYSIS.  THE INITIAL FLOW VARIABLES FOR THE BOATTAIL FLOW CAN BE SPECIFIED OR COMPUTED FROM A UNIFORM
     FLOW ANALYSIS.  THE NOZZLE AND BOATTAIL GEOMETRIES CAN BE SPECIFIED AS CONICAL, SECOND-ORDER POLYNOMIAL OR TABULAR CONTOUR
     THE ANALYSIS WAS BASED ON THE GOVERNING GAS DYNAMIC RELATIONS FOR A ROTATIONAL FLOW OF A FROZEN OR EQUILIBRIUM GAS MIXTURE


JOB TITLE

       SAMPLE CASE 2      BOATTAIL ANALYSIS
```

Figure F-2. Output for Sample Case 2.

BOATTAIL FLOW FIELD ANALYSIS

```
NP    = 30        TO  =  600.00 R       XE =  10.00 IN      G =        1.40
IC    =  2        PO  =   12.00 PSIA    YT =   8.00 IN      RG =      53.30 (FT-LBF)/(LBM-R)
IVL   =  0        PAMB =   0.00 PSIA    RO =   5.00 IN      NGAS =     1
IWALL =  2                                                   TOL  =     .00100
DELTA =1.00                                                  SPACE =    .5
KWRITE =2
```

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A UNIFORM FLOW WITH A MACH NUMBER OF 1.40

I	X	Y	M	P	Q	TH	U	V	R	T
1	0.000	8.000	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
2	.167	8.170	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
3	.333	8.340	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
4	.500	8.510	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
5	.667	8.680	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
6	.833	8.851	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
7	1.000	9.021	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
8	1.167	9.191	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
9	1.333	9.361	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
10	1.500	9.531	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
11	1.667	9.701	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
12	1.833	9.871	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
13	2.000	10.041	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
14	2.167	10.211	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
15	2.333	10.381	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
16	2.500	10.552	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
17	2.667	10.722	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
18	2.833	10.892	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
19	3.000	11.062	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
20	3.167	11.232	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
21	3.333	11.402	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
22	3.500	11.572	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
23	3.667	11.742	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
24	3.833	11.912	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
25	4.000	12.082	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
26	4.167	12.253	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
27	4.333	12.423	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
28	4.500	12.593	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
29	4.667	12.763	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
30	4.833	12.933	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0

BOATTAIL GEOMETRY

THE BOATTAIL LENGTH IS XE =   10.0000 IN.

THE THROAT WALL CONTOUR IS A CIRCULAR ARC WITH YT = 8.000 IN. THETAT =   2.000 DEGREES. RU = 5.000 IN.

Figure F-2. Output for Sample Case 2 (Continued).

THE SUPERSONIC WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   2.000 DEGREES AND YE = 7.000 IN.

I	J	X	Y	M	P	Q	TH	U	V	R	T	IC/THRSTV
2	2	.345	7.991	1.473	3.401	1476.0	-2.1318	1475.0	-54.9	2.1957E-02	418.5	1.7
3	3	.696	7.977	1.480	3.364	1481.3	-2.4039	1480.0	-62.1	2.1785E-02	417.2	4.0
4	4	1.049	7.961	1.488	3.327	1486.6	-2.6772	1485.0	-69.4	2.1616E-02	415.9	6.6
5	5	1.403	7.944	1.495	3.292	1491.7	-2.9516	1489.8	-76.8	2.1450E-02	414.6	9.5
6	6	1.759	7.925	1.503	3.257	1496.8	-3.2273	1494.4	-84.3	2.1288E-02	413.3	12.7
7	7	2.117	7.904	1.510	3.223	1501.7	-3.5040	1498.9	-91.8	2.1130E-02	412.1	16.0
8	8	2.476	7.881	1.517	3.190	1506.6	-3.7818	1503.3	-99.4	2.0975E-02	410.9	19.7
9	9	2.837	7.856	1.524	3.158	1511.3	-4.0607	1507.5	-107.0	2.0823E-02	409.7	23.6
10	10	3.200	7.829	1.531	3.126	1515.9	-4.3406	1511.6	-114.7	2.0675E-02	408.5	27.7
11	11	3.564	7.801	1.537	3.096	1520.5	-4.6215	1515.5	-122.5	2.0530E-02	407.4	32.1
12	12	3.930	7.770	1.544	3.066	1524.9	-4.9034	1519.3	-130.3	2.0388E-02	406.3	36.6
13	13	4.297	7.738	1.551	3.037	1529.3	-5.1862	1523.0	-138.2	2.0249E-02	405.2	41.5
14	14	4.666	7.703	1.557	3.008	1533.5	-5.4699	1526.5	-146.2	2.0114E-02	404.1	46.5
15	15	5.036	7.667	1.563	2.981	1537.7	-5.7545	1529.9	-154.2	1.9982E-02	403.0	51.8
16	16	5.408	7.629	1.569	2.954	1541.7	-6.0400	1533.2	-162.2	1.9853E-02	402.0	57.2
17	17	5.781	7.588	1.575	2.928	1545.7	-6.3262	1536.3	-170.3	1.9727E-02	400.9	62.9
18	18	6.156	7.546	1.581	2.902	1549.6	-6.6133	1539.3	-178.5	1.9605E-02	400.0	68.8
19	19	6.532	7.501	1.587	2.878	1553.4	-6.9011	1542.1	-186.6	1.9485E-02	399.0	74.9
20	20	6.909	7.455	1.593	2.854	1557.0	-7.1897	1544.8	-194.9	1.9369E-02	398.0	81.2
21	21	7.288	7.406	1.598	2.830	1560.6	-7.4790	1547.3	-203.1	1.9257E-02	397.1	87.7
22	22	7.669	7.355	1.604	2.808	1564.1	-7.7689	1549.7	-211.4	1.9148E-02	396.2	94.3
23	23	8.050	7.302	1.609	2.786	1567.4	-8.0595	1551.9	-219.8	1.9042E-02	395.3	101.1
24	24	8.433	7.247	1.614	2.765	1570.7	-8.3508	1554.0	-228.1	1.8939E-02	394.5	108.1
25	25	8.818	7.189	1.619	2.745	1573.8	-8.6426	1556.0	-236.5	1.8840E-02	393.6	115.3
26	26	9.203	7.130	1.624	2.726	1576.9	-8.9350	1557.7	-244.9	1.8745E-02	392.8	122.7
27	27	9.590	7.068	1.628	2.707	1579.8	-9.2279	1559.4	-253.3	1.8653E-02	392.1	130.2
28	28	9.979	7.004	1.633	2.689	1582.6	-9.5212	1560.8	-261.8	1.8565E-02	391.3	137.8

Figure F-2. Output for Sample Case 2 (Continued).

I	J	X	Y	M	P	W	TH	U	V	R	T	IC/THRSTV
29	29	10.000	7.000	1.633	2.687	1582.9	-9.5373	1561.0	-262.3	1.8556E-02	391.3	138.2

Figure F-2. Output for Sample Case 2 (Concluded).

3. Sample Case 3

Sample case 3 illustrates a complete nozzle-base-boattail flow field analysis for the combined nozzle and boattail contours specified in sample cases 1 and 2. Consequently, 11 is placed in the first two columns of the title card. The total thrust on the assembly is the sum of the thrusts acting on the nozzle, the boattail, and the base region. For this sample case, KWRITN = 2 and KWRITB = 2. This print option dictates that wall points only will be written out. The following list presents the data deck for sample case 3.

```
11   SAMPLE CASE 3    NOZZLE-BASE-BOATTAIL ANALYSIS
 $INFO KWRITN=2, KWRITB=2, YEB=5.2 $
```

Figure F-3 presents the output for sample case 3. The first page of the output lists the program abstract and the job title for the run. The second page presents the nozzle flow field analysis data and initial-value line. The third page presents the nozzle wall geometric specification, and the third and fourth pages present the nozzle flow field solution. At the end of the flow field data, -2 appears in the last column denoting the nozzle exit lip point. The number of internally crossing characteristics is zero.

The three pages following the nozzle flow field results present the boattail flow field results. The first page lists input variables, initial-value line data, and geometric parameters. The next page presents the boattail flow field solution. For this sample case, KWRITB = 2, thus, only wall point solutions are printed out. Each right-running characteristic begins on the initial-value line and extends to the boattail wall, where the thrust is computed. This procedure continues until a right-running characteristic passes the exit lip point. Then the inverse wall point routine computes the boattail exit lip point properties.

The base pressure and the contribution to the thrust from that pressure are then calculated. The exit lip point properties of the nozzle and the boattail, as well as the base region properties, are printed out on the last page. The total thrust, which is written out on the last line, is the sum of the nozzle, boattail, and base thrusts.

Sample case 3 required 13 seconds of central processor time on the CDC 6500 computer.

ANALYSIS AND DESIGN OF SUPERSONIC NOZZLE-BASE-BOATTAIL COMBINATIONS

ABSTRACT

THIS PROGRAM WAS PRODUCED AT THE PURDUE UNIVERSITY THERMAL SCIENCE AND PROPULSION CENTER BY J. G. ALLMAN AS A PART OF THE REQUIREMENTS OF AF CONTRACT NUMBER F33615-73-C-2010. THE CONTRACT WAS SPONSORED BY THE AERO PROPULSION LABORATORY WRIGHT PATTERSON AFB, OHIO. PRINCIPAL INVESTIGATOR FOR PURDUE UNIVERSITY WAS PROFESSOR JOE D. HOFFMAN.

THE EQUATIONS OF MOTION FOR AN AXISYMMETRIC SUPERSONIC FLOW ARE SOLVED USING A NUMERICAL METHOD OF CHARACTERISTICS HAVING SECOND-ORDER ACCURACY. THE INITIAL FLOW VARIABLES FOR THE NOZZLE FLOW FIELD CAN BE SPECIFIED OR COMPUTED FROM KLIEGELS TRANSONIC FLOW ANALYSIS. THE INITIAL FLOW VARIABLES FOR THE BOATTAIL FLOW CAN BE SPECIFIED OR COMPUTED FROM A UNIFORM FLOW ANALYSIS. THE NOZZLE AND BOATTAIL GEOMETRIES CAN BE SPECIFIED AS CONICAL, SECOND-ORDER POLYNOMIAL OR TABULAR CONTOURS. THE ANALYSIS WAS BASED ON THE GOVERNING GAS DYNAMIC RELATIONS FOR A ROTATIONAL FLOW OF A FROZEN OR EQUILIBRIUM GAS MIXTURE.

JOB TITLE

SAMPLE CASE NUMBER 3 NOZZLE-BASE-BOATTAIL ANALYSIS

Figure F-3. Output for Sample Case 3.

NOZZLE FLOW FIELD ANALYSIS

```
NP =    11         TO =   6000.00 R          XE =10.000 IN        G =       1.20
IC =     2         PO =   1000.00 PSIA       YE =  5.000 IN       RG =     60.00 (FT-LBF)/(LBM-R)
IVL =    0         PAMB =    0.00 PSIA       RU =  3.000 IN       NGAS =     1
IWALL = 2                                    RD =   .500 IN       RATIOI =5.00
NPW =   22                                   YT =  1.000 IN       RATIOW =5.00
KWRITE =2                                                         TOL =     .0010
```

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A KLIEGEL V=0 LINE

I	X	Y	M	P	Q	TH	U	V	R	T	WDOT	THRUST	ISP
1	0.000	1.000	1.082	514.66	3816.6	-.000	3816.6	-.0	2.2997E-01	5371.2	19.221	3896.821	202.733
2	.012	.961	1.075	519.00	3793.7	.004	3793.7	.3	2.3156E-01	5378.7	17.744	3596.924	202.709
3	.025	.914	1.067	523.76	3768.6	.010	3768.6	.7	2.3335E-01	5386.9	16.054	3253.909	202.684
4	.040	.857	1.058	528.90	3741.6	.019	3741.6	1.3	2.3525E-01	5395.7	14.143	2866.122	202.660
5	.057	.790	1.049	534.33	3713.0	.031	3713.0	2.0	2.3726E-01	5404.9	12.014	2434.523	202.637
6	.075	.710	1.040	539.92	3683.6	.043	3683.6	2.8	2.3933E-01	5414.3	9.696	1964.469	202.615
7	.094	.614	1.031	545.49	3654.3	.054	3654.3	3.4	2.4139E-01	5423.5	7.248	1468.370	202.596
8	.114	.499	1.022	550.76	3626.6	.059	3626.6	3.7	2.4333E-01	5432.2	4.786	969.651	202.581
9	.132	.361	1.015	555.39	3602.4	.054	3602.4	3.4	2.4503E-01	5439.8	2.511	508.582	202.569
10	.146	.197	1.009	558.82	3584.3	.034	3584.3	2.1	2.4629E-01	5445.4	.745	150.840	202.561
11	.151	0.000	1.007	560.23	3577.0	0.000	3577.0	0.0	2.4681E-01	5447.7	0.000	0.000	0.000

INITIAL-VALUE LINE PERFORMANCE PARAMETERS

THE NOZZLE MASS FLOW RATE IS   19.221 (LBM/SEC)

THE START LINE THRUST IS   3896.8 LBF, AND THE START LINE ISP IS 202.733 (LBF-SEC)/LBM

THE ONE-DIMENSIONAL MASS FLOW RATE IS   19.261 (LBM/SEC), AND THE DISCHARGE COEFFICIENT IS .9979

THE ONE-DIMENSIONAL THRUST IS   3897.0 LBF, AND THE ONE-DIMENSIONAL SPECIFIC IMPULSE IS 202.323 (LBF-SEC/LBM)

THE SPECIFIC IMPULSE EFFICIENCY IS 1.0020

Figure F-3. Output for Sample Case 3 (Continued).

THE SUPERSONIC WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT = 30.000 DEGREES AND YE = 5.000 IN

J	I	X	Y	M	P	Q	TH	U	V	R	T	MDT/LRC	F/LRC	THRUST	ISP	IC/EF
1	12	.005	1.000	1.107	499.791	3895.2	.543	3895.0	36.9	2.2441E-01	5345.0	17.744	3596.9	3896.9	202.737	2
1	13	.010	1.000	1.132	485.317	3972.1	1.130	3971.3	78.3	2.1899E-01	5318.9	17.946	3637.9	3897.1	202.749	2
1	14	.015	1.000	1.156	470.917	4049.0	1.763	4047.1	124.6	2.1356E-01	5292.3	16.242	3696.0	3897.5	202.770	2
1	15	.021	1.000	1.182	456.380	4127.2	2.447	4123.4	176.2	2.0805E-01	5264.7	18.491	3748.9	3898.2	202.804	2
1	16	.028	1.001	1.208	441.520	4207.7	3.185	4201.2	233.8	2.0239E-01	5235.7	18.713	3794.5	3899.1	202.850	2
2	17	.035	1.001	1.236	426.189	4291.5	3.982	4281.1	298.0	1.9652E-01	5204.9	17.796	3609.5	3900.3	202.912	2
2	18	.042	1.002	1.264	410.783	4376.6	4.843	4361.0	369.5	1.9058E-01	5173.1	18.040	3660.0	3901.8	202.991	2
2	19	.050	1.003	1.294	394.877	4465.5	5.772	4442.8	449.1	1.8441E-01	5139.2	18.269	3708.0	3903.7	203.090	2
2	20	.059	1.003	1.326	378.388	4558.9	6.775	4527.1	537.8	1.7797E-01	5102.7	18.468	3753.9	3906.0	203.211	2
2	21	.068	1.005	1.360	361.185	4658.0	7.858	4614.3	636.8	1.7120E-01	5063.3	18.698	3798.3	3908.8	203.357	2
3	22	.078	1.006	1.397	343.203	4763.5	9.027	4704.5	747.4	1.6407E-01	5020.4	17.862	3637.5	3912.2	203.531	2
3	23	.089	1.008	1.434	325.382	4870.3	10.289	4792.0	869.9	1.5[illegible]94E-01	4975.9	18.118	3688.2	3916.1	203.734	2
3	24	.101	1.010	1.475	306.916	4983.6	11.652	4880.9	1006.5	1.[illegible]948E-01	4927.7	18.345	3737.7	3920.6	203.970	2
3	25	.114	1.013	1.519	287.758	5104.5	13.123	4971.2	1159.0	1.4167E-01	4875.0	18.564	3785.7	3925.8	204.241	2
3	26	.127	1.016	1.567	267.788	5234.6	14.712	5063.0	1329.4	1.3342E-01	4816.9	18.774	3832.5	3931.7	204.548	2
4	27	.141	1.020	1.619	247.492	5371.9	16.427	5152.6	1519.2	1.2494E-01	4754.0	18.058	3698.3	3938.3	204.893	2
4	28	.157	1.025	1.673	227.484	5513.0	18.279	5234.8	1729.1	1.1647E-01	4687.6	18.293	3751.5	3945.7	205.275	2
4	29	.173	1.031	1.732	207.096	5663.8	20.278	5312.7	1963.0	1.0771E-01	4614.7	18.517	3803.0	3953.8	205.696	2
4	30	.191	1.038	1.797	186.404	5825.5	22.437	5384.5	2223.4	9.8663E-02	4534.3	18.729	3852.1	3962.5	206.152	2
5	31	.209	1.046	1.869	165.641	5998.5	24.767	5446.7	2513.0	8.9420E-02	4445.8	18.097	3743.2	3971.9	206.641	2
5	32	.229	1.056	1.946	145.657	6177.7	27.283	5490.4	2831.8	8.0339E-02	4351.3	18.329	3798.2	3981.8	207.156	2
5	33	.250	1.067	2.030	126.088	6368.3	30.000	5515.1	3184.2	7.1244E-02	4247.5	18.546	3850.0	3992.1	207.691	2
6	34	.294	1.092	2.046	122.646	6403.8	29.933	5549.6	3195.4	6.9619E-02	4228.0	19.225	4014.0	4013.5	208.802	1
7	35	.400	1.153	2.082	115.203	6482.7	29.769	5627.2	3218.7	6.6081E-02	4184.1	19.226	4065.4	4064.6	211.461	2
8	36	.521	1.222	2.123	107.223	6571.0	29.583	5714.4	3244.0	6.2244E-02	4134.3	19.227	4122.8	4121.8	214.435	2
9	37	.657	1.299	2.169	98.854	6668.3	29.372	5811.1	3270.6	5.8169E-02	4078.6	19.228	4185.8	4184.5	217.700	2
10	38	.807	1.383	2.220	90.367	6772.6	29.138	5915.5	3297.7	5.3977E-02	4018.0	19.228	4253.0	4251.5	221.186	2
11	39	.969	1.473	2.274	82.064	6881.2	28.885	6025.1	3324.0	4.9813E-02	3953.9	19.228	4322.7	4321.0	224.800	2
12	40	1.113	1.552	2.321	75.358	6974.4	28.658	6120.0	3344.8	4.6397E-02	3898.0	19.229	4382.3	4380.3	227.886	2
13	41	1.237	1.619	2.361	70.115	7051.3	28.464	6198.9	3360.7	4.3692E-02	3851.4	19.229	4431.3	4429.0	230.418	2
14	42	1.351	1.681	2.397	65.639	7120.2	28.283	6270.1	3373.7	4.1354E-02	3809.4	19.229	4475.1	4472.4	232.675	1
15	43	1.463	1.741	2.432	61.585	7185.3	28.106	6338.0	3385.0	3.9215E-02	3769.1	19.230	4516.4	4513.3	234.806	1
16	44	1.574	1.800	2.466	57.641	7248.2	27.928	6404.1	3394.8	3.7217E-02	3729.9	19.230	4556.2	4552.7	236.855	1
17	45	1.685	1.859	2.499	54.370	7309.2	27.750	6468.5	3403.2	3.5346E-02	3691.7	19.230	4594.6	4590.7	238.830	1
18	46	1.795	1.917	2.532	51.207	7367.1	27.574	6530.3	3410.2	3.3624E-02	3655.0	19.231	4631.0	4626.7	240.704	1
19	47	1.902	1.972	2.563	48.348	7421.6	27.402	6588.9	3415.7	3.2053E-02	3620.2	19.231	4665.2	4660.5	242.463	1

Figure F-3. Output for Sample Case 3 (Continued).

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PROPULSION NOZZLE STUDIES. VOLUME II. DESIGN OF MAXIMUM THRUST --ETC(U)
MAR 77 J G ALLMAN, J D HOFFMAN F33615-73-C-2010

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J	I	X	Y	M	P	Q	TH	U	V	R	T	MDT/LHC	F/LHC	THRUST	ISP	IC/EFF
20	48	2.005	2.025	2.592	45.796	7472.3	27.237	6643.8	3419.8	3.0636E-02	3587.6	19.231	4696.9	4691.8	244.091	1
21	49	2.104	2.076	2.620	43.447	7520.7	27.076	6696.5	3423.2	2.9321E-02	3556.3	19.231	4726.6	4721.1	245.616	1
22	50	2.658	2.354	2.767	33.064	7749.8	26.171	6964.3	3422.5	2.3365E-02	3396.2	19.232	4874.3	4868.9	253.304	2
23	51	3.162	2.597	2.885	26.457	7942.1	25.334	7178.3	3398.4	1.9409E-02	3271.5	19.232	4986.9	4981.6	259.168	2
24	52	3.674	2.835	2.992	21.631	8097.8	24.474	7370.2	3354.7	1.6414E-02	3162.8	19.231	5084.1	5079.0	264.236	2
25	53	4.208	3.073	3.090	17.972	8234.0	23.562	7547.5	3291.5	1.4067E-02	3066.2	19.230	5171.4	5166.5	268.788	2
26	54	4.789	3.320	3.183	15.067	8357.8	22.557	7718.4	3206.0	1.2147E-02	2977.0	19.228	5253.2	5248.6	273.058	2
27	55	5.433	3.580	3.271	12.734	8471.0	21.424	7885.6	3094.2	1.0559E-02	2894.3	19.226	5331.2	5326.9	277.135	2
28	56	6.149	3.852	3.355	10.670	8573.2	20.145	8048.8	2952.6	9.2552E-03	2818.7	19.224	5405.6	5401.8	281.031	2
29	57	6.959	4.137	3.433	9.377	8665.3	18.673	8209.2	2774.3	8.1836E-03	2750.0	19.220	5477.5	5474.3	284.803	2
30	58	7.883	4.434	3.505	8.196	8746.4	16.960	8366.0	2551.3	7.3153E-03	2688.8	19.216	5547.1	5544.6	288.461	2
31	59	8.945	4.738	3.567	7.287	8815.1	14.953	8516.6	2274.6	6.6334E-03	2636.5	19.211	5613.8	5612.4	291.985	2
32	60	10.000	5.000	3.609	6.727	8860.9	12.921	8636.5	1981.4	6.2058E-03	2601.6	19.206	5669.2	5668.5	294.906	-2

0 CHARACTERISTICS CROSSED IN THE NOZZLE.

Figure F-3. Output for Sample Case 3 (Continued).

```
BOATTAIL FLOW FIELD ANALYSIS

NP    = 30        TO   =  600.00 R         XE =  10.00 IN       G =        1.40
IC    = 2         PO   =   12.00 PSIA      YT =   8.00 IN       RG =      53.30 (FT-LBF)/(LBM-R)
IVL   = 0         PAMB =    0.00 PSIA      RD =   5.00 IN       NGAS =     1
IWALL = 2                                                       TOL  =    .00100
DELTA =1.00                                                     SPACE =    .5
KWRITE =2
```

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A UNIFORM FLOW WITH A MACH NUMBER OF 1.40

I	X	Y	M	P	Q	TH	U	V	R	T
1	0.000	8.000	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
2	.167	8.170	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
3	.333	8.340	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
4	.500	8.510	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
5	.667	8.680	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
6	.833	8.851	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
7	1.000	9.021	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
8	1.167	9.191	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
9	1.333	9.361	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
10	1.500	9.531	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
11	1.667	9.701	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
12	1.833	9.871	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
13	2.000	10.041	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
14	2.167	10.211	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
15	2.333	10.381	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
16	2.500	10.552	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
17	2.667	10.722	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
18	2.833	10.892	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
19	3.000	11.062	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
20	3.167	11.232	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
21	3.333	11.402	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
22	3.500	11.572	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
23	3.667	11.742	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
24	3.833	11.912	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
25	4.000	12.082	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
26	4.167	12.253	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
27	4.333	12.423	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
28	4.500	12.593	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
29	4.667	12.763	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
30	4.833	12.933	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0

BOATTAIL GEOMETRY

THE BOATTAIL LENGTH IS XE =   10.0000 IN.

THE THROAT WALL CONTOUR IS A CIRCULAR ARC WITH YT =   8.000 IN. THETAT =   2.000 DEGREES. RU =   5.000 IN.

Figure F-3. Output for Sample Case 3 (Continued).

THE SUPERSONIC WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT = 2.000 DEGREES AND YE = 5.200 IN.

I	J	X	Y	M	P	Q	TH	U	V	R	T	IC/THRSTV
2	2	.345	7.990	1.485	3.341	1484.6	-2.4970	1483.2	-64.7	2.1681E-02	416.4	1.7
3	3	.702	7.971	1.518	3.185	1507.3	-3.5332	1504.4	-92.9	2.0952E-02	410.7	4.8
4	4	1.066	7.946	1.551	3.036	1529.4	-4.5886	1524.5	-122.4	2.0244E-02	405.1	8.8
5	5	1.438	7.912	1.584	2.893	1551.0	-5.6619	1543.5	-153.0	1.9559E-02	399.6	13.8
6	6	1.816	7.871	1.616	2.756	1572.1	-6.7518	1561.2	-184.8	1.8895E-02	394.1	19.5
7	7	2.202	7.822	1.649	2.626	1592.6	-7.8571	1577.7	-217.7	1.8253E-02	388.7	26.1
8	8	2.595	7.764	1.681	2.502	1612.6	-8.9768	1592.9	-251.6	1.7632E-02	383.3	33.4
9	9	2.995	7.696	1.713	2.384	1632.1	-10.1095	1606.7	-286.5	1.7034E-02	378.1	41.4
10	10	3.402	7.620	1.745	2.272	1651.0	-11.2538	1619.2	-322.2	1.6459E-02	372.9	50.0
11	11	3.816	7.533	1.776	2.166	1669.2	-12.4086	1630.2	-358.7	1.5906E-02	367.8	59.1
12	12	4.236	7.436	1.807	2.066	1686.9	-13.5722	1639.8	-395.9	1.5378E-02	362.9	68.8
13	13	4.664	7.328	1.838	1.972	1703.9	-14.7433	1647.8	-433.6	1.4875E-02	358.1	78.9
14	14	5.099	7.209	1.867	1.883	1720.2	-15.9202	1654.2	-471.8	1.4396E-02	353.4	89.4
15	15	5.540	7.078	1.896	1.801	1735.7	-17.1014	1659.0	-510.4	1.3945E-02	349.0	100.2
16	16	5.987	6.935	1.924	1.725	1750.5	-18.2850	1662.1	-549.2	1.3521E-02	344.7	111.3
17	17	6.442	6.780	1.951	1.655	1764.4	-19.4693	1663.5	-588.1	1.3125E-02	340.6	122.6
18	18	6.902	6.612	1.977	1.591	1777.4	-20.6524	1663.2	-626.9	1.2759E-02	336.8	134.0
19	19	7.368	6.430	2.001	1.533	1789.3	-21.8324	1661.0	-665.4	1.2425E-02	333.2	145.6
20	20	7.841	6.236	2.023	1.481	1800.2	-23.0073	1657.0	-703.6	1.2124E-02	330.0	157.3
21	21	8.318	6.027	2.042	1.436	1809.9	-24.1751	1651.1	-741.2	1.1859E-02	327.1	169.0
22	22	8.800	5.805	2.060	1.397	1818.2	-25.3337	1643.4	-778.0	1.1632E-02	324.5	180.8
23	23	9.287	5.568	2.074	1.366	1825.1	-26.4809	1633.6	-813.8	1.1445E-02	322.5	192.4
24	24	9.778	5.317	2.085	1.343	1830.3	-27.6145	1621.8	-848.4	1.1304E-02	320.9	204.0
25	25	10.000	5.200	2.090	1.334	1832.3	-28.1203	1616.0	-863.6	1.1253E-02	320.3	209.2

Figure F-3. Output for Sample Case 3 (Continued).

```
NOZZLE AND BOATTAIL EXIT LIP POINT CONDITIONS

                X       Y       M       P       Q        TH        U       V          R          T    THRUST

NOZZLE     10.000   5.000   3.609   6.727  8660.9   12.9213   8636.5  1981.4  6.2058E-03   2601.6   5668.5

BOATTAIL            5.200   2.090   1.334  1832.3  -28.1203   1616.0  -863.6  1.1253E-02    320.3    209.2

THE BASE PRESSURE IS      .961 (LBF/IN2) AND THE BASE THRUST IS      6.16 (LBF)
THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5883.9 (LBF)
```

Figure F-3. Output for Sample Case 3 (Concluded).

4. Sample Case 4

Sample case 4 illustrates a nozzle optimization using Newton's method. The nozzle gas dynamic model and geometry are specified by the program default valves (see Sample Case 1). To reduce the output, KWRITN = 3. The locations of the initial-value points are specified by NPN = 9 and RATIØI = 4.0. The locations of the points on the initial expansion contour are specified by NPW = 12 and RATIØW = 3.0. The initial estimate for the throat attachment angle is ANSTRT(1) = 25.0. To minimize the flow field output during the optimization procedure, KWRITØ = 3. The nozzle attachment angle perturbation DA = 2.0. For a nozzle optimization, IØP = 1 specifies the geometric model and N = 2 specifies a two-dimensional optimization. The following list presents the data deck for sample case 4.

```
22   SAMPLE CASE 4    NOZZLE OPTIMIZATION
 $INFO   KWRITN=3, NPN=9, RATIØI=4.0, NPW=12, RATIØW=3.0,
   ANSTRT(1)=25.0, KWRITØ=3, DA=2.0 $
```

The first page of the output prints the program abstract and the job title. The second page presents the input data, the initial-value line, and the initial-value line performance parameters. For this sample case, KWRITN = 3 and KWRITØ = 3 to minimize output. A secondary-start line is computed at the minimum attachment angle, ANMIN(1) = 20.0, which is printed out on the next page (see the discussion in Section II). The optimization parameters follow on the next page, which lists the type of geometry (IØP = 1), print option (KWRITØ = 3), convergence tolerance (TØLØ = 0.001), optimization variable space (N = 2), optimization method (IMETH = 3), and maximum number of base point moves (ITERØ = 10). The initial estimate for the throat attachment angle is 25.0 deg, and for the exit lip radius is 5.0 in. These values, and the minimum and maximum values of those variables, are listed for reference.

The next three pages present the results of the optimization. On the first of those three pages, a heading is printed out specifying the optimization method. The next three lines present the results of the nozzle flow field analysis for the initial nozzle contour. The output for each nozzle flow field analysis is reduced to three lines. The first line specifies the nozzle geometry, the second line presents the flow properties at the nozzle exit lip point, and the third line is the crossing characteristics count. After the analysis of the initial nozzle contour, an optimization base point data line is written out stating that for the base point (0 step), THETA = 25.0 deg, YE = 5.000 in., and the nozzle thrust is 5644.65 lbf.

Newton's method requires the values of the thrust for the base point and five neighboring points. The nozzles corresponding to these five points are analyzed, and the results presented in the next five groups of three lines each. From that data, Newton's method constructs the base point move and analyzes the corresponding new base point nozzle.

The next three lines present the results of that analysis. An optimization base point data line is then written out presenting the base point data for that step (1 step).

The above procedure is repeated for a total of three base point moves, at which time the optimization procedure has converged. The last line of output shows that the optimum nozzle contour has a throat attachment angle of 32.729 deg, an exit lip point radius of 5.244 in., and a thrust of 5679.81 lbf.

Sample case 4 required 80 seconds of central processor time on the CDC 6500 computer.

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                ANALYSIS AND DESIGN OF SUPERSONIC NOZZLE-BASE-BOATTAIL COMBINATIONS

ABSTRACT

     THIS PROGRAM WAS PRODUCED AT THE PURDUE UNIVERSITY THERMAL SCIENCE AND PROPULSION CENTER BY J. G. ALLMAN AS A PART OF THE
     REQUIREMENTS OF AF CONTRACT NUMBER F33615-73-C-2010.  THE CONTRACT WAS SPONSORED BY THE AERO PROPULSION LABORATORY WRIGHT
     PATTERSON AFB, OHIO.  PRINCIPAL INVESTIGATOR FOR PURDUE UNIVERSITY WAS PROFESSOR JOE D. HOFFMAN.

     THE EQUATIONS OF MOTION FOR AN AXISYMMETRIC SUPERSONIC FLOW ARE SOLVED USING A NUMERICAL METHOD OF CHARACTERISTICS HAVING
     SECOND-ORDER ACCURACY.  THE INITIAL FLOW VARIABLES FOR THE NOZZLE FLOW FIELD CAN BE SPECIFIED OR COMPUTED FROM KLIEGELS
     TRANSONIC FLOW ANALYSIS.  THE INITIAL FLOW VARIABLES FOR THE BOATTAIL FLOW CAN BE SPECIFIED OR COMPUTED FROM A UNIFORM
     FLOW ANALYSIS.  THE NOZZLE AND BOATTAIL GEOMETRIES CAN BE SPECIFIED AS CONICAL, SECOND-ORDER POLYNOMIAL OR TABULAR CONTOUR
     THE ANALYSIS WAS BASED ON THE GOVERNING GAS DYNAMIC RELATIONS FOR A ROTATIONAL FLOW OF A FROZEN OR EQUILIBRIUM GAS MIXTURE


JOB TITLE

       SAMPLE CASE 4      NOZZLE OPTIMIZATION
```

Figure F-4. Output for Sample Case 4.

NOZZLE FLOW FIELD ANALYSIS

NP = 9	TO = 6000.00 R	XE =10.000 IN	G = 1.20
IC = 2	PO = 1000.00 PSIA	YE = 5.000 IN	RG = 60.00 (FT-LBF)/(LBM-R)
IVL = 0	PAMB = 0.00 PSIA	RU = 3.000 IN	NGAS = 1
IWALL = 2		RD = .500 IN	RATIOI =4.00
NPW = 12		YT = 1.000 IN	RATIOW =3.00
KWRITE =3			TOL = .0010

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A KLIEGEL V=0 LINE

I	X	Y	M	P	Q	TH	U	V	R	T	WDOT	THRUST	ISP
1	0.000	1.000	1.082	514.66	3816.6	-.000	3816.6	-.0	2.2997E-01	5371.2	19.221	3896.783	202.734
2	.017	.943	1.072	520.79	3784.3	.006	3784.3	.4	2.3224E-01	5381.8	17.117	3469.632	202.701
3	.036	.875	1.061	527.38	3749.5	.017	3749.5	1.1	2.3469E-01	5393.1	14.715	2982.163	202.668
4	.057	.791	1.049	534.29	3713.2	.031	3713.2	2.0	2.3725E-01	5404.8	12.029	2437.534	202.638
5	.080	.688	1.038	541.26	3676.5	.046	3676.5	2.9	2.3983E-01	5416.5	9.119	1847.587	202.612
6	.103	.564	1.027	547.95	3641.4	.057	3641.4	3.6	2.4230E-01	5427.6	6.114	1238.623	202.590
7	.126	.411	1.017	553.89	3610.2	.057	3610.2	3.6	2.4448E-01	5437.4	3.260	660.301	202.574
8	.144	.226	1.010	558.36	3586.7	.038	3586.7	2.4	2.4612E-01	5444.7	.984	199.281	202.563
9	.151	0.000	1.007	560.23	3577.0	0.000	3577.0	0.0	2.4681E-01	5447.7	0.000	0.000	0.000

INITIAL-VALUE LINE PERFORMANCE PARAMETERS

THE NOZZLE MASS FLOW RATE IS 19.221 (LBM/SEC)

THE START LINE THRUST IS 3896.8 LBF, AND THE START LINE ISP IS 202.734 (LBF-SEC)/LBM

THE ONE-DIMENSIONAL MASS FLOW RATE IS 19.261 (LBM/SEC), AND THE DISCHARGE COEFFICIENT IS .9979

THE ONE-DIMENSIONAL THRUST IS 3896.9 LBF, AND THE ONE-DIMENSIONAL SPECIFIC IMPULSE IS 202.322 (LBF-SEC/LBM)

THE SPECIFIC IMPULSE EFFICIENCY IS 1.0020

THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT = 20.000 DEGREES AND YE = 5.000 IN

Figure F-4. Output for Sample Case 4 (Continued).

SECONDARY START LINE

J	I	X	Y	M	P	Q	TH	U	V	R	T	MDT/LRC	F/LRC	THRUST	ISP	IC/EF
21	1	.171	1.030	1.723	210.083	5640.6	20.000	5300.6	1929.3	1.0901E-01	4625.3	18.311	3755.7			
21	2	.190	1.025	1.729	208.190	5657.9	20.017	5316.1	1936.7	1.0814E-01	4620.6	18.905	3893.3			
21	3	.256	1.007	1.744	203.391	5696.1	20.254	5343.9	1971.9	1.0602E-01	4604.0	17.848	3695.3			
21	4	.331	.988	1.763	197.431	5743.2	20.471	5380.5	2008.5	1.0341E-01	4582.0	16.700	3479.9			
21	5	.415	.967	1.786	190.250	5800.3	20.646	5427.8	2045.2	1.0026E-01	4554.3	15.484	3251.6			
21	6	.505	.946	1.813	181.966	5867.5	20.759	5486.6	2079.7	9.6600E-02	4520.9	14.243	3018.0			
21	7	.600	.926	1.844	172.916	5942.9	20.798	5555.7	2110.1	9.2575E-02	4482.8	13.033	2788.9			
21	8	.681	.909	1.872	165.085	6010.6	20.775	5619.7	2132.0	8.9056E-02	4448.9	12.072	2606.1			
21	9	.747	.896	1.897	158.518	6069.3	20.714	5676.9	2146.8	8.6077E-02	4419.8	11.323	2462.7			
21	10	.808	.884	1.920	152.522	6124.3	20.635	5731.4	2158.3	8.3338E-02	4392.4	10.670	2337.0			
21	11	.868	.873	1.943	146.630	6179.6	20.522	5787.4	2166.4	8.0631E-02	4364.5	10.064	2219.6			
21	12	.927	.862	1.967	140.817	6235.4	20.385	5844.9	2172.0	7.7945E-02	4335.9	9.491	2108.0			
21	13	.985	.852	1.991	135.242	6290.4	20.231	5902.3	2175.2	7.5351E-02	4307.6	8.961	2004.0			
21	14	1.040	.842	2.015	129.962	6343.7	20.062	5958.8	2176.1	7.2878E-02	4279.9	8.478	1906.4			
21	15	1.093	.833	2.038	124.775	6397.5	19.848	6017.5	2172.1	7.0433E-02	4251.7	8.035	1820.3			
21	16	1.386	.786	2.169	99.236	6674.0	18.584	6326.0	2127.0	5.8263E-02	4087.8	5.938	1391.9			
21	17	1.619	.749	2.278	81.674	6893.8	17.298	6582.0	2049.8	4.9567E-02	3954.6	4.641	1116.2			
21	18	1.837	.716	2.360	67.833	7091.3	15.996	6816.7	1954.2	4.2482E-02	3832.2	3.665	901.7			
21	19	2.055	.681	2.481	56.357	7277.7	14.665	7040.6	1842.5	3.6416E-02	3714.2	2.878	722.9			
21	20	2.288	.643	2.587	46.369	7462.9	13.245	7264.4	1709.9	3.0966E-02	3593.8	2.210	566.2			
21	21	2.537	.600	2.696	37.824	7645.3	11.762	7484.8	1558.5	2.6145E-02	3472.1	1.649	430.6			
21	22	2.814	.549	2.813	30.349	7830.7	10.170	7707.6	1382.7	2.1777E-02	3344.7	1.170	311.5			
21	23	3.131	.487	2.941	23.846	8020.6	8.449	7933.5	1178.5	1.7830E-02	3209.7	.769	208.6			
21	24	3.501	.409	3.084	18.234	8216.8	6.573	8162.8	940.6	1.4281E-02	3064.2	.445	122.9			
21	25	3.938	.309	3.244	13.526	8417.5	4.543	8391.0	666.8	1.1170E-02	2906.4	.205	57.5			
21	26	4.460	.178	3.427	9.694	8619.8	2.377	8612.4	357.5	8.5162E-03	2731.4	.055	15.8			
21	27	5.109	0.000	3.654	6.595	8827.3	0.000	8827.3	0.0	6.2835E-03	2519.1	0.000	0.0			

Figure F-4. Output for Sample Case 4 (Continued).

```
J I   X     Y     M     P     Q     TH    U     V     R     T   MDT/LHC  F/LHC  THRUST  ISP  IC/EFF

      OPTIMIZATION PARAMETERS

      IOP    = 1           N      = 2
      KWRITO = 3           IMETH  = 3
      TOLO   =.00100       ITERO  =10

         START      MINIMUM     MAXIMUM
      1  25.00      20.00       45.00      NOZZLE ATTACHMENT ANGLE
      2   5.00       4.00        7.00      NOZZLE EXIT RADIUS
```

Figure F-4. Output for Sample Case 4 (Continued).

```
NOZZLE OPTIMIZATION IS BEING CARRIED OUT USING NEWTONS METHOD.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  25.000 DEGREES AND YE =  5.000 IN
23 50 10.000  5.000  3.853    4.336  9083.5  18.849  8596.3  2934.7  4.3386E-03  2398.8   19.237   5642.0    5644.6  293.669    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     AT THE   0 STEP   THETA = 25.000    YE =  5.000    THRUST =     5644.65

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  27.000 DEGREES AND YE =  5.000 IN
23 51 10.000  5.000  3.759    5.139  8998.8  16.589  8624.3  2569.2  4.9857E-03  2473.9   19.219   5655.7    5660.5  294.494    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  25.000 DEGREES AND YE =  5.200 IN
22 49 10.000  5.200  3.950    3.654  9166.1  20.920  8562.0  3272.6  3.7715E-03  2325.1   19.250   5640.0    5641.5  293.504    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  27.000 DEGREES AND YE =  5.200 IN
22 50 10.000  5.200  3.856    4.310  9088.1  18.718  8607.5  2916.5  4.3134E-03  2398.3   19.225   5651.8    5657.4  294.331    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  23.000 DEGREES AND YE =  5.000 IN
23 51 10.000  5.000  3.945    3.687  9160.4  20.975  8553.4  3279.0  3.8024E-03  2327.2   19.252   5624.3    5625.2  292.658    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  25.000 DEGREES AND YE =  4.800 IN
23 50 10.000  4.800  3.759    5.142  8997.2  16.727  8616.5  2589.5  4.9309E-03  2472.8   19.234   5643.7    5645.2  293.697    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  34.864 DEGREES AND YE =  4.877 IN
23 53 10.000  4.877  3.319   11.647  8526.8   4.745  8497.6   705.4  9.8122E-03  2848.8   19.124   5645.3    5657.8  294.352    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     AT THE   1 STEP   THETA = 34.864    YE =  4.877    THRUST =     5657.79

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  36.864 DEGREES AND YE =  4.877 IN
23 54 10.000  4.877  3.218   14.106  8401.0   1.654  8397.5   242.5  1.1505E-02  2942.5   19.099   5619.3    5635.0  293.166    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  34.864 DEGREES AND YE =  5.077 IN
23 53 10.000  5.077  3.402    9.971  8624.6   7.079  8558.8  1062.9  8.6239E-03  2774.8   19.120   5654.1    5669.6  294.965    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  36.864 DEGREES AND YE =  5.077 IN
23 54 10.000  5.077  3.296   12.167  8498.8   4.011  8478.0   594.4  1.0174E-02  2870.1   19.095   5635.5    5653.8  294.147    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.864 DEGREES AND YE =  4.877 IN
23 53 10.000  4.877  3.418    9.665  8644.0   7.659  8566.9  1152.1  8.4034E-03  2760.4   19.156   5661.2    5671.2  295.053    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  34.864 DEGREES AND YE =  4.677 IN
23 53 10.000  4.677  3.240   13.533  8428.6   2.395  8421.3   352.3  1.1116E-02  2921.9   19.125   5628.2    5638.9  293.372    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.392 DEGREES AND YE =  5.063 IN
23 53 10.000  5.063  3.522    7.958  8760.1  10.457  8614.7  1589.9  7.1519E-03  2670.6   19.159   5666.2    5678.4  295.426    -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     AT THE   2 STEP   THETA = 32.392    YE =  5.063    THRUST =     5678.41

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  34.392 DEGREES AND YE =  5.063 IN
```

Figure F-4. Output for Sample Case 4 (Continued).

```
23 53 10.000  5.063  3.420    9.631  8646.0   7.616  8569.7  1145.9  8.3788E-03  2758.6   19.128   5657.2   5671.9  295.086      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.392 DEGREES AND YE =  5.263 IN
22 52 10.000  5.263  3.604    6.830  8847.5  12.716  8630.5  1947.5  6.3007E-03  2601.6   19.109   5672.0   5679.7  295.493      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  34.392 DEGREES AND YE =  5.263 IN
22 52 10.000  5.263  3.499    8.308  8734.8   9.920  8604.2  1504.7  7.4116E-03  2690.2   19.168   5668.8   5677.1  295.355      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.392 DEGREES AND YE =  5.063 IN
23 52 10.000  5.063  3.623    6.595  8866.4  13.120  8635.0  2012.7  6.1220E-03  2585.3   19.182   5666.3   5675.9  295.292      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.392 DEGREES AND YE =  4.863 IN
23 53 10.000  4.863  3.436    9.355  8664.0   8.164  8576.2  1230.3  8.1785E-03  2745.1   19.162   5662.9   5672.3  295.109      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

     THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.729 DEGREES AND YE =  5.244 IN
22 52 10.000  5.244  3.578    7.164  8820.7  12.042  8626.6  1840.2  6.5545E-03  2623.1   19.186   5672.1   5679.8  295.498      -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.

        AT THE   3 STEP   THETA = 32.729   YE =  5.244   THRUST =    5679.81
```

Figure F-4. Output for Sample Case 4 (Concluded).

5. Sample Case 5

Sample case 5 illustrates a nozzle-base-boattail assembly optimization. The maximum thrust contour is approached in two steps. First, a two-dimensional optimization is performed. This is accomplished by requiring the width of the base region to be the minimum allowable value ($y_{eb} = y_{en} + \Delta y_b$), which leaves two parameters (θ_{an} and y_{en}) unconstrained. This optimization converges quickly because of the relatively few function evaluations required for a base point move. After convergence, the optimization problem is redefined so that the boattail and nozzle exit lip points are independent, thus yielding a three-dimensional optimization problem. From the preliminary two-dimensional optimization, a good initial approximation to the maximum thrust nozzle-base-boattail contour is available, and the three-dimensional optimization converges in one pass.

The general operating conditions considered in sample case 5 are the same as those in sample case 3. The boattail outer radius YTB = 5.3 in., the radius of curvature of the boattail initial expansion contour is the program default value RDB = 5.0 in., and the boattail contour is conical (IWALLB = 1). Output is minimized by setting KWRITN, KWRITB, and KWRITØ equal to 3. Computation time for each analysis is kept small by setting NPN = 9 and NPB = 40. The nozzle has 12 inverse wall points (NPW = 12) spanning the minimum attachment angle [ANMIN(1) = 20.0 deg.]. The initial-value line points and the wall points are spaced according to RATIØI = 3.0, RATIØW = 3.0, and SPACE = 0.6.

Now the optimization employs Newton's method (IMETH = 3) to calculate the maximum thrust contours to within a relative tolerance of TØLØ = 0.0001 or within a given number of iterations ITERØ = 10. An initial estimate for the nozzle geometry is given by specifying ANSTRT(1) = 30.0 deg and ANSTRT(2) = 4.0 in. The maximum values of θ_{an}, y_{en}, θ_{ab}, and y_{eb} are specified as ANMAX(1) = 40.0 deg, 5.0 in., 20.0 deg, and 5.0 in., respectively. The minimum values of those parameters are specified as the program default values. For a nozzle-base-boattail optimization, IØP = 3 specifies the type of geometry, and N = 3 specifies that a three-dimensional optimization is to be performed (i.e., the unconstrained geometric variables are the nozzle throat attachment angle θ_{an}, the nozzle exit lip radius y_{en}, and the boattail exit lip radius y_{eb}. The θ_{an} perturbation DA = 2.0 deg, the y_{en} perturbation DB = 0.1 y_{tn}, and the y_{eb} perturbation DD = 0.2 y_{tn}. The minimum annular base width DBAS = 0.1 in.

The following list presents the data deck for sample case 5.

```
21   SAMPLE CASE 5   NOZZLE-BASE-BOATTAIL OPTIMIZATION
 $INFO   KWRITN=3, KWRITB=3, KWRITØ=3, YTB=5.3, IWALLB=1,
   NPN=9, NPW=12, NPB=40, RATIØI=3.0, RATIØW=3.0, SPACE=0.6,
   ANSTRT(1)=30.0, 4.0, ANMAX(1)=40.0, 5.0, 20.0, 5.0,
   IMETH=3, ITERØ=10, TØLØ=0.0001, IØP=3, N=3,
   DA=2.0, DB=0.1, DBAS=0.1 $
```

Figure F-5 presents the output for sample case 5. The first page presents the program abstract and the job title. The second page presents the nozzle flow field analysis data, the nozzle initial-value line, and the nozzle initial-value line performance parameters. The third page presents the nozzle secondary start line. The optimization parameters, the initial values of the nozzle geometric parameters, and the minimum and maximum values of the nozzle and boattail geometric parameters are presented on the fourth page. The fifth page presents the boattail flow field analysis data and the boattail initial-value line.

The results of the optimization procedure are presented on pages six to ten of Fig. F-5. The first line on page six identifies the optimization procedure as Newton's method. The next group of seven lines summarizes the results of the combined nozzle-base-boattail flow field analysis for the initial contour. The first line specifies the nozzle geometry, the second line present the flow properties at the nozzle exit lip point, and the third line presents the crossing characteristics counter. The fourth line presents the boattail geometry, the fifth line presents the flow field properties at the boattail exit lip point, and the six line present the base region pressure and thrust. Line seven presents the total nozzle-base-boattail thrust. Each flow field analysis for a particular nozzle-base-boattail configuration generates a corresponding set of seven lines of output.

After the analysis of the initial nozzle-base-boattail configuration, an optimization base point data line is written out summarizing the geometry and the corresponding thrust. The initial contour is designated the 0 step.

The two-dimensional optimization, in which the boattail exit lip point is constrained to be a fixed distance from the nozzle exit lip point, then commences. This procedure is analogous to the nozzle optimization discussed in sample case 4. Newton's method requires the values of thrust at the base point and five neighboring points. Those five points are the first five points after the 0 step data line. From that data, Newton's method constructs the base point move and analyzes the new base point configuration. The sixth set of data after the 0 step presents the results of that analysis. An optimization base point data line is then written out presenting the base point data for that step (1 step).

The above constrained boattail optimization is repeated for a total of three base point moves. The final results are printed out on the eight page of Fig. F-7, as the seven lines immediately preceding the optimization data line for step 3. At this time, a full three-dimensional optimization is initiated in which the conical boattail moves independently of the nozzle exit lip point. The initial contour for the three-dimensional optimization is chosen to be the nozzle contour obtained in the constrained boattail optimization described above, and a boattail exit lip point moved outward from the nozzle exit lip point by

twice the minimum separation distance Δy_b. That contour is then analyzed and printed out as the next seven lines of output. The corresponding optimization base point data line is identified as step 0 for the three-dimensional optimization.

For the three-dimensional optimization, Newton's method requires the values of the thrust for the base point configuration and nine neighboring points. Those results are presented on the last three pages of Fig. F-5. From that data, Newton's method constructs the base point move and analyzes the corresponding new base point configuration. The final group of seven lines on the last page of Fig. F-5 presents the results of that analysis. The last line on Fig. F-5 presents the optimization base point data line for that step, step 1. For the present sample case, the results of step 1 satisfy the convergence criteria and the program stops.

Sample case 5 required 240 seconds of central processor time on the CDC 6500 computer.

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          ANALYSIS AND DESIGN OF SUPERSONIC NOZZLE-BASE-BOATTAIL COMBINATIONS

ABSTRACT

     THIS PROGRAM WAS PRODUCED AT THE PURDUE UNIVERSITY THERMAL SCIENCE AND PROPULSION CENTER BY J. G. ALLMAN AS A PART OF THE
     REQUIREMENTS OF AF CONTRACT NUMBER F33615-73-C-2010.  THE CONTRACT WAS SPONSORED BY THE AERO PROPULSION LABORATORY WRIGHT
     PATTERSON AFB, OHIO.  PRINCIPAL INVESTIGATOR FOR PURDUE UNIVERSITY WAS PROFESSOR JOE D. HOFFMAN.

     THE EQUATIONS OF MOTION FOR AN AXISYMMETRIC SUPERSONIC FLOW ARE SOLVED USING A NUMERICAL METHOD OF CHARACTERISTICS HAVING
     SECOND-ORDER ACCURACY.  THE INITIAL FLOW VARIABLES FOR THE NOZZLE FLOW FIELD CAN BE SPECIFIED OR COMPUTED FROM KLIEGELS
     TRANSONIC FLOW ANALYSIS.  THE INITIAL FLOW VARIABLES FOR THE BOATTAIL FLOW CAN BE SPECIFIED OR COMPUTED FROM A UNIFORM
     FLOW ANALYSIS.  THE NOZZLE AND BOATTAIL GEOMETRIES CAN BE SPECIFIED AS CONICAL, SECOND-ORDER POLYNOMIAL OR TABULAR CONTOUR
     THE ANALYSIS WAS BASED ON THE GOVERNING GAS DYNAMIC RELATIONS FOR A ROTATIONAL FLOW OF A FROZEN OR EQUILIBRIUM GAS MIXTURE


JOB TITLE

     SAMPLE CASE 5       NOZZLE-BASE-BOATTAIL OPTIMIZATION
```

Figure F-5. Output for Sample Case 5.

NOZZLE FLOW FIELD ANALYSIS

NP = 9	TO = 6000.00 R	XE =10.000 IN	G = 1.20
IC = 2	PO = 1000.00 PSIA	YE = 5.000 IN	RG = 60.00 (FT-LBF)/(LBM-R)
IVL = 0	PAMB = 0.00 PSIA	RU = 3.000 IN	NGAS = 1
IWALL = 2		RD = .500 IN	RATIOI =3.00
NPW = 12		YT = 1.000 IN	RATIOW =3.00
KWRITE =3			TOL = .0010

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A KLIEGEL V=0 LINE

I	X	Y	M	P	Q	TH	U	V	R	T	WDOT	THRUST	ISP
1	0.000	1.000	1.082	514.66	3816.6	-.000	3816.6	-.0	2.2997E-01	5371.2	19.221	3896.780	202.734
2	.020	.932	1.070	521.92	3778.3	.008	3778.3	.5	2.3267E-01	5383.8	16.714	3387.858	202.695
3	.041	.853	1.058	529.27	3739.6	.020	3739.6	1.3	2.3539E-01	5396.3	14.000	2837.272	202.659
4	.064	.760	1.046	536.50	3701.5	.035	3701.5	2.3	2.3807E-01	5408.5	11.127	2254.750	202.629
5	.087	.652	1.034	543.40	3665.3	.050	3665.3	3.2	2.4062E-01	5420.1	8.183	1657.871	202.604
6	.110	.525	1.024	549.67	3632.4	.058	3632.4	3.7	2.4293E-01	5430.4	5.310	1075.667	202.584
7	.130	.377	1.016	554.94	3604.7	.055	3604.7	3.4	2.4487E-01	5439.1	2.733	553.666	202.570
8	.145	.203	1.009	558.72	3584.8	.035	3584.8	2.2	2.4626E-01	5445.2	.795	160.953	202.562
9	.151	0.000	1.007	560.23	3577.0	0.000	3577.0	0.0	2.4681E-01	5447.7	0.000	0.000	0.000

INITIAL-VALUE LINE PERFORMANCE PARAMETERS

THE NOZZLE MASS FLOW RATE IS 19.221 (LBM/SEC)

THE START LINE THRUST IS 3896.8 LBF, AND THE START LINE ISP IS 202.734 (LBF-SEC)/LBM

THE ONE-DIMENSIONAL MASS FLOW RATE IS 19.261 (LBM/SEC), AND THE DISCHARGE COEFFICIENT IS .9979

THE ONE-DIMENSIONAL THRUST IS 3896.9 LBF, AND THE ONE-DIMENSIONAL SPECIFIC IMPULSE IS 202.321 (LBF-SEC/LBM)

THE SPECIFIC IMPULSE EFFICIENCY IS 1.0020

THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT = 20.000 DEGREES AND YE = 5.000 IN

Figure F-5. Output for Sample Case 5 (Continued).

SECONDARY START LINE

J	I	X	Y	M	P	ρ	TH	U	V	R	T	MDT/LRC	F/LRC	THRUST	ISP	TC/EF
21	1	.171	1.030	1.724	209.919	5642.1	20.000	5301.8	1929.7	1.0894E-01	4624.7	17.937	3683.3			
21	2	.210	1.019	1.734	206.708	5669.4	20.103	5324.0	1948.6	1.0749E-01	4615.3	18.562	3832.7			
21	3	.278	1.001	1.750	201.576	5710.3	20.334	5354.4	1984.3	1.0523E-01	4597.3	17.500	3630.0			
21	4	.353	.983	1.769	195.514	5758.4	20.532	5392.6	2019.6	1.0257E-01	4574.8	16.375	3419.1			
21	5	.433	.963	1.791	188.558	5814.2	20.683	5439.4	2053.6	9.9508E-02	4547.8	15.233	3204.6			
21	6	.516	.944	1.817	180.898	5876.8	20.776	5494.6	2084.6	9.6120E-02	4516.8	14.108	2992.5			
21	7	.600	.926	1.845	172.820	5944.4	20.807	5556.7	2111.6	9.2522E-02	4482.9	13.033	2789.1			
21	8	.673	.911	1.870	165.741	6005.7	20.786	5614.8	2131.3	8.9339E-02	4452.4	12.164	2623.8			
21	9	.735	.898	1.893	159.676	6059.8	20.737	5667.2	2145.7	8.6590E-02	4425.7	11.463	2489.9			
21	10	.792	.887	1.914	154.035	6111.4	20.669	5718.0	2157.1	8.4016E-02	4400.2	10.841	2370.3			
21	11	.850	.876	1.937	148.356	6164.3	20.568	5771.4	2165.7	8.1412E-02	4373.5	10.249	2255.8			
21	12	.909	.865	1.960	142.534	6219.7	20.438	5828.2	2171.9	7.8728E-02	4345.1	9.669	2142.9			
21	13	.969	.855	1.985	136.670	6276.9	20.284	5887.6	2176.0	7.6008E-02	4315.4	9.105	2032.5			
21	14	1.031	.844	2.011	130.789	6335.6	20.101	5949.7	2177.4	7.3263E-02	4284.5	8.562	1925.3			
21	15	1.093	.833	2.039	124.718	6397.9	19.858	6017.5	2173.3	7.0409E-02	4251.2	8.038	1821.1			
21	16	1.387	.786	2.169	99.151	6674.9	16.591	6326.6	2128.0	5.8223E-02	4087.1	5.940	1392.3			
21	17	1.619	.750	2.278	81.602	6894.6	17.305	6582.5	2050.9	4.9532E-02	3953.9	4.642	1116.6			
21	18	1.837	.716	2.380	67.783	7092.0	16.005	6817.1	1955.4	4.2456E-02	3831.7	3.668	902.3			
21	19	2.056	.682	2.481	56.330	7276.1	14.676	7040.6	1843.9	3.6402E-02	3713.8	2.862	723.7			
21	20	2.287	.644	2.586	46.383	7462.6	13.261	7263.6	1711.9	3.0974E-02	3593.9	2.215	567.6			
21	21	2.540	.600	2.697	37.721	7647.6	11.757	7487.2	1558.3	2.6066E-02	3470.4	1.646	430.0			
21	22	2.817	.550	2.814	30.289	7832.2	10.171	7709.2	1383.1	2.1741E-02	3343.5	1.170	311.4			
21	23	3.131	.488	2.942	23.829	8021.1	8.460	7933.8	1180.1	1.7820E-02	3209.4	.771	209.1			
21	24	3.496	.411	3.083	18.268	8215.6	6.602	8161.1	944.5	1.4303E-02	3065.3	.449	124.0			
21	25	3.936	.311	3.243	13.546	8416.6	4.566	8389.9	670.0	1.1182E-02	2907.2	.207	58.2			
21	26	4.467	.178	3.429	9.652	8622.3	2.357	8615.0	354.6	8.4891E-03	2728.8	.054	15.5			
21	27	5.113	0.000	3.655	6.588	8827.8	0.000	8827.8	0.0	6.2775E-03	2518.6	0.000	0.0			

Figure F-5. Output for Sample Case 5 (Continued).

```
J  I   X      Y      M       P       Q      TH      U      V       R        T    MDT/LHC   F/LHC   THRUST   ISP   TC/EFF

        OPTIMIZATION PARAMETERS

        IOP     = 3          N      = 3
        K*PITO  = 3          IMETH  = 3
        TOLO    =.00010      ITERO  =10
        DEAS    = .10

            START       MINIMUM     MAXIMUM
        1   30.00        20.00       40.00      NOZZLE ATTACHMENT ANGLE
        2    4.00         4.00        5.00      NOZZLE EXIT RADIUS
        3   15.00         0.00       20.00      BOATTAIL ATTACHMENT ANGLE
        4    4.00         4.00        5.00      BOATTAIL EXIT RADIUS
```

Figure F-5. Output for Sample Case 5 (Continued).

BOATTAIL FLOW FIELD ANALYSIS

```
NP    = 40        TO   =  600.00 R      XE =  10.00 IN    G =       1.40
IC    = 2         PO   =   12.00 PSIA   YT =   5.30 IN    RG =     53.30 (FT-LBF)/(LBM-R)
IVL   = 0         PAMB =    0.00 PSIA   RD =   5.00 IN    NGAS =    1
IWALL = 2                                                 TOL  =    .00100
DELTA =1.00                                               SPACE =   .6
KWRITE =3
```

INITIAL-VALUE LINE

THE INITIAL-VALUE LINE IS A UNIFORM FLOW WITH A MACH NUMBER OF 1.40

I	X	Y	M	P	Q	TH	U	V	R	T
1	0.000	5.300	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
2	.150	5.453	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
3	.300	5.606	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
4	.450	5.759	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
5	.600	5.912	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
6	.750	6.065	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
7	.900	6.219	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
8	1.050	6.372	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
9	1.200	6.525	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
10	1.350	6.678	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
11	1.500	6.831	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
12	1.650	6.984	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
13	1.800	7.137	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
14	1.950	7.290	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
15	2.100	7.443	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
16	2.250	7.596	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
17	2.400	7.749	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
18	2.550	7.903	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
19	2.700	8.056	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
20	2.850	8.209	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
21	3.000	8.362	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
22	3.150	8.515	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
23	3.300	8.668	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
24	3.450	8.821	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
25	3.600	8.974	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
26	3.750	9.127	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
27	3.900	9.280	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
28	4.050	9.434	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
29	4.200	9.587	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
30	4.350	9.740	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
31	4.500	9.893	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
32	4.650	10.046	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
33	4.800	10.199	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
34	4.950	10.352	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
35	5.100	10.505	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
36	5.250	10.658	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
37	5.400	10.811	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
38	5.550	10.964	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
39	5.700	11.119	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0
40	5.850	11.271	1.400	3.77	1424.2	0.000	1424.2	0.0	2.3636E-02	431.0

Figure F-5. Output for Sample Case 5 (Continued).

```
A  3-DIMENSIONAL OPTIMIZATION IS BEING CARRIED OUT USING NEWTONS METHOD.

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.000 DEGREES AND YE =  4.000 IN
23 52 10.000  4.000  3.196   14.692  8373.6   1.392  8371.9   171.6  1.1901E-02  2962.9   19.183   5596.9   5597.7  291.226      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   7.059 DEGREES AND YE =  4.100 IN
31 31 10.000  4.100  1.502    3.258 1496.5  -7.0578  1485.1  -183.9  2.1302E-02   413.3    106.1
   THE BASE PRESSURE IS    2.721 (LBF/IN2) AND THE BASE THRUST IS      6.92 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5710.7 (LBF)

        AT THE   0 STEP   THETA = 30.000   YE =  4.000   THETA =  7.059   YE =  4.100   THRUST =    5710.71

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.000 DEGREES AND YE =  4.000 IN
23 53 10.000  4.000  3.111   17.249  8262.5  -1.383  8259.9  -208.0  1.3598E-02  3044.3   19.144   5566.0   5574.1  290.001      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   7.059 DEGREES AND YE =  4.100 IN
31 31 10.000  4.100  1.502    3.258 1496.5  -7.0578  1485.1  -183.9  2.1302E-02   413.3    106.1
   THE BASE PRESSURE IS    2.736 (LBF/IN2) AND THE BASE THRUST IS      6.96 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5687.2 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.000 DEGREES AND YE =  4.100 IN
23 52 10.000  4.100  3.232   13.738  8418.8   2.565  8410.4   376.8  1.1255E-02  2929.4   19.180   5611.0   5613.3  292.039      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   6.459 DEGREES AND YE =  4.200 IN
32 32 10.000  4.200  1.491    3.313 1488.6  -6.4574  1479.1  -167.4  2.1555E-02   415.3    100.3
   THE BASE PRESSURE IS    2.763 (LBF/IN2) AND THE BASE THRUST IS      7.20 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5720.8 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  32.000 DEGREES AND YE =  4.100 IN
23 53 10.000  4.100  3.148   16.101  8310.9   -.206  8310.8   -29.9  1.2841E-02  3009.2   19.149   5586.0   5593.4  291.003      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   6.459 DEGREES AND YE =  4.200 IN
32 32 10.000  4.200  1.491    3.313 1488.6  -6.4574  1479.1  -167.4  2.1555E-02   415.3    100.3
   THE BASE PRESSURE IS    2.783 (LBF/IN2) AND THE BASE THRUST IS      7.26 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5700.9 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.000 DEGREES AND YE =  4.000 IN
23 52 10.000  4.000  3.275   12.664  8472.2   4.046  8451.1   597.7  1.0521E-02  2889.0   19.200   5609.8   5608.8  291.904      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   7.059 DEGREES AND YE =  4.100 IN
31 31 10.000  4.100  1.502    3.258 1496.5  -7.0578  1485.1  -183.9  2.1302E-02   413.3    106.1
   THE BASE PRESSURE IS    2.699 (LBF/IN2) AND THE BASE THRUST IS      6.87 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5721.8 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.000 DEGREES AND YE =  3.900 IN
24 53 10.000  3.900  3.156   15.855  8321.4    .217  8321.3    31.5  1.2679E-02  3001.2   19.136   5570.4   5580.0  290.307      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   8.264 DEGREES AND YE =  3.900 IN
31 31 10.000  3.900  1.512    3.211 1503.3  -8.2648  1487.7  -216.1  2.1083E-02   411.5    116.6
   THE BASE PRESSURE IS    2.693 (LBF/IN2) AND THE BASE THRUST IS      0.00 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5696.6 (LBF)
 THIS ASSEMBLY IS A PHYSICAL IMPOSSIBILITY

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.597 DEGREES AND YE =  4.235 IN
23 52 10.000  4.235  3.346   11.072  8559.0   6.000  8512.1   894.7  9.4091E-03  2824.2   19.203   5635.9   5635.9  293.216      -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.653 DEGREES AND YE =  4.335 IN
32 32 10.000  4.335  1.483    3.351 1483.2  -5.6540  1476.0  -146.1  2.1727E-02   416.6     91.6
   THE BASE PRESSURE IS    2.729 (LBF/IN2) AND THE BASE THRUST IS      7.35 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5734.9 (LBF)
```

Figure F-5. Output for Sample Case 5 (Continued).

```
          AT THE  1 STEP   THETA = 28.597   YE =  4.235   THETA =  5.653   YE =  4.335   THRUST =     5734.89

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.597 DEGREES AND YE =  4.235 IN
23 52 10.000  4.235  3.260   13.027  8454.1   3.329  8439.8   491.0  1.0769E-02  2903.2   19.178   5624.5   5627.2  292.761       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.653 DEGREES AND YE =  4.335 IN
32 32 10.000  4.335  1.483     3.351 1483.2  -5.6540  1476.0  -146.1  2.1727E-02   416.6     91.6
  THE BASE PRESSURE IS    2.779 (LBF/IN2) AND THE BASE THRUST IS      7.48 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5726.3 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.597 DEGREES AND YE =  4.335 IN
23 52 10.000  4.335  3.388   10.227  8606.7   7.159  8541.6  1072.8  8.8088E-03  2786.3   19.204   5644.5   5645.1  293.691       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.058 DEGREES AND YE =  4.435 IN
32 32 10.000  4.435  1.476     3.382 1478.7  -5.0571  1472.9  -130.3  2.1873E-02   417.8     84.6
  THE BASE PRESSURE IS    2.721 (LBF/IN2) AND THE BASE THRUST IS      7.50 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.1 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.597 DEGREES AND YE =  4.335 IN
23 52 10.000  4.335  3.300   12.072  8503.8   4.500  8477.6   667.2  1.0109E-02  2866.0   19.160   5636.2   5638.9  293.372       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.058 DEGREES AND YE =  4.435 IN
32 32 10.000  4.435  1.476     3.382 1478.7  -5.0571  1472.9  -130.3  2.1873E-02   417.8     84.6
  THE BASE PRESSURE IS    2.782 (LBF/IN2) AND THE BASE THRUST IS      7.66 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5731.2 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  26.597 DEGREES AND YE =  4.235 IN
23 52 10.000  4.235  3.433    9.416  8658.8   8.541  8562.8  1286.0  8.2268E-03  2746.8   19.221   5636.1   5634.1  293.118       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.653 DEGREES AND YE =  4.335 IN
32 32 10.000  4.335  1.483     3.351 1483.2  -5.6540  1476.0  -146.1  2.1727E-02   416.6     91.6
  THE BASE PRESSURE IS    2.661 (LBF/IN2) AND THE BASE THRUST IS      7.16 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5732.8 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.597 DEGREES AND YE =  4.135 IN
23 52 10.000  4.135  3.305   11.974  8509.0   4.837  8478.7   717.5  1.0041E-02  2862.0   19.200   5625.1   5625.1  292.651       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   6.851 DEGREES AND YE =  4.135 IN
31 31 10.000  4.135  1.500     3.267 1495.2  -6.8504  1484.5  -178.3  2.1343E-02   413.6    104.1
  THE BASE PRESSURE IS    2.685 (LBF/IN2) AND THE BASE THRUST IS      0.00 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5729.2 (LBF)
 THIS ASSEMBLY IS A PHYSICAL IMPOSSIBILITY

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.552 DEGREES AND YE =  4.349 IN
23 52 10.000  4.349  3.396   10.074  8618.0   7.381  8546.6  1107.1  8.6994E-03  2779.2   19.204   5645.7   5646.2  293.752       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.974 DEGREES AND YE =  4.449 IN
32 32 10.000  4.449  1.476     3.387 1478.0  -4.9731  1472.4  -128.1  2.1894E-02   417.9     83.6
  THE BASE PRESSURE IS    2.718 (LBF/IN2) AND THE BASE THRUST IS      7.51 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.3 (LBF)

          AT THE  2 STEP   THETA = 28.552   YE =  4.349   THETA =  4.974   YE =  4.449   THRUST =     5737.30

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.552 DEGREES AND YE =  4.349 IN
23 52 10.000  4.349  3.308   11.896  8513.3   4.727  8484.3   701.5  9.9863E-03  2858.9   19.181   5638.0   5640.7  293.464       -2
       0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.974 DEGREES AND YE =  4.449 IN
32 32 10.000  4.449  1.476     3.387 1478.0  -4.9731  1472.4  -128.1  2.1894E-02   417.9     83.6
  THE BASE PRESSURE IS    2.781 (LBF/IN2) AND THE BASE THRUST IS      7.69 (LBF)
  THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5731.9 (LBF)
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Figure F-5. Output for Sample Case 5 (Continued).

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   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.552 DEGREES AND YE =  4.449 IN
23 52 10.000  4.449  3.439     9.294  8667.2   8.532  8571.3  1285.9  8.1372E-03  2741.3   19.205   5652.2   5653.5  294.128      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.381 DEGREES AND YE =  4.549 IN
33 33 10.000  4.549  1.464     3.441 1470.3  -4.3798  1466.0  -112.3  2.2144E-02   419.8     76.0
   THE BASE PRESSURE IS     2.725 (LBF/IN2) AND THE BASE THRUST IS      7.70 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.1 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.552 DEGREES AND YE =  4.449 IN
23 52 10.000  4.449  3.349    11.007  8562.9   5.892  8517.6   879.1  9.3622E-03  2821.5   19.163   5647.2   5650.3  293.962      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.381 DEGREES AND YE =  4.549 IN
33 33 10.000  4.549  1.464     3.441 1470.3  -4.3798  1466.0  -112.3  2.2144E-02   419.8     76.0
   THE BASE PRESSURE IS     2.798 (LBF/IN2) AND THE BASE THRUST IS      7.91 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5734.1 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  26.552 DEGREES AND YE =  4.349 IN
23 52 10.000  4.349  3.485     8.542  8717.0   9.901  8597.2  1498.8  7.5894E-03  2701.4   19.222   5643.2   5641.9  293.527      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.974 DEGREES AND YE =  4.449 IN
32 32 10.000  4.449  1.476     3.387 1478.0  -4.9731  1472.4  -128.1  2.1894E-02   417.9     83.6
   THE BASE PRESSURE IS     2.640 (LBF/IN2) AND THE BASE THRUST IS      7.30 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5732.8 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.552 DEGREES AND YE =  4.249 IN
23 52 10.000  4.249  3.354    10.909  8568.4   6.223  8517.9   928.9  9.2938E-03  2817.1   19.203   5637.3   5637.4  293.292      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   6.167 DEGREES AND YE =  4.249 IN
32 32 10.000  4.249  1.488     3.326 1486.7  -6.1661  1478.1  -159.7  2.1615E-02   415.7     97.2
   THE BASE PRESSURE IS     2.700 (LBF/IN2) AND THE BASE THRUST IS      0.00 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5734.6 (LBF)
 THIS ASSEMRLY IS A PHYSICAL IMPOSSIBILITY

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.411     9.802  8634.9   7.759  8555.8  1165.7  8.5041E-03  2766.3   19.203   5649.1   5650.1  293.952      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.674 DEGREES AND YE =  4.499 IN
32 32 10.000  4.499  1.472     3.404 1475.6  -4.6732  1470.7  -120.2  2.1973E-02   418.5     79.8
   THE BASE PRESSURE IS     2.719 (LBF/IN2) AND THE BASE THRUST IS      7.60 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.5 (LBF)

        AT THE   3 STEP    THETA = 28.710    YE =  4.399    THETA =  4.674    YE =  4.499    THRUST =     5737.47

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.411     9.802  8634.9   7.759  8555.8  1165.7  8.5041E-03  2766.3   19.203   5649.1   5650.1  293.952      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461     3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS     2.767 (LBF/IN2) AND THE BASE THRUST IS     15.65 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.6 (LBF)

        AT THE   0 STEP    THETA = 28.710    YE =  4.399    THLTA =  4.082    YE =  4.599    THRUST =     5737.64

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.322    11.593  8529.8   5.100  8496.0   758.3  9.7749E-03  2846.5   19.160   5641.9   5645.0  293.666      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461     3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS     2.836 (LBF/IN2) AND THE BASE THRUST IS     16.03 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5732.9 (LBF)
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Figure F-5. Output for Sample Case 5 (Continued).

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   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.499 IN
23 52 10.000  4.499  3.454    9.041  8683.9   8.908  8579.1  1344.7  7.9530E-03  2728.5   19.204   5654.9   5656.7  294.297     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461    3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS    2.724 (LBF/IN2) AND THE BASE THRUST IS     7.79 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5736.4 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.411    9.802  8634.9   7.759  8555.8  1165.7  8.5041E-03  2746.3   19.203   5649.1   5650.1  293.952     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   2.905 DEGREES AND YE =  4.799 IN
33 33 10.000  4.799  1.446    3.533 1457.3  -2.9035  1455.4   -73.8  2.2563E-02   423.0     54.5
   THE BASE PRESSURE IS    2.834 (LBF/IN2) AND THE BASE THRUST IS    32.76 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.3 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.710 DEGREES AND YE =  4.499 IN
23 52 10.000  4.499  3.363   10.723  8579.3   6.265  8528.1   936.2  9.1614E-03  2809.1   19.181   5650.3   5653.9  294.150     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461    3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS    2.802 (LBF/IN2) AND THE BASE THRUST IS     8.01 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5733.8 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  26.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.500    8.300  8734.0  10.280  8593.8  1558.7  7.4102E-03  2698.1   19.220   5646.3   5645.5  293.712     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461    3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS    2.685 (LBF/IN2) AND THE BASE THRUST IS    15.18 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5732.6 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.299 IN
23 52 10.000  4.299  3.369   10.617  8585.4   6.603  8528.5   987.2  9.0871E-03  2804.2   19.202   5641.5   5641.9  293.526     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   4.082 DEGREES AND YE =  4.599 IN
33 33 10.000  4.599  1.461    3.458 1467.8  -4.0810  1464.1  -104.5  2.2224E-02   420.4     71.9
   THE BASE PRESSURE IS    2.807 (LBF/IN2) AND THE BASE THRUST IS    23.54 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.4 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.411    9.802  8634.9   7.759  8555.8  1165.7  8.5041E-03  2746.3   19.203   5649.1   5650.1  293.952     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   5.269 DEGREES AND YE =  4.399 IN
32 32 10.000  4.399  1.479    3.371 1480.3  -5.2676  1474.1  -135.9  2.1820E-02   417.3     87.1
   THE BASE PRESSURE IS    2.689 (LBF/IN2) AND THE BASE THRUST IS     -.00 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.2 (LBF)
 THIS ASSEMBLY IS A PHYSICAL IMPOSSIBILITY

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  30.710 DEGREES AND YE =  4.399 IN
23 52 10.000  4.399  3.322   11.593  8529.8   5.100  8496.0   758.3  9.7749E-03  2846.5   19.100   5641.9   5645.0  293.686     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   2.905 DEGREES AND YE =  4.799 IN
33 33 10.000  4.799  1.446    3.533 1457.3  -2.9035  1455.4   -73.8  2.2563E-02   423.0     54.5
   THE BASE PRESSURE IS    2.905 (LBF/IN2) AND THE BASE THRUST IS    33.58 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5733.0 (LBF)

   THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.710 DEGREES AND YE =  4.499 IN
23 52 10.000  4.499  3.454    9.041  8683.9   8.908  8579.1  1344.7  7.9530E-03  2728.5   19.204   5654.9   5656.7  294.297     -2
      0 CHARACTERISTICS CROSSED IN THE NOZZLE.
   THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   2.905 DEGREES AND YE =  4.799 IN
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Figure F-5. Output for Sample Case 5 (Continued).

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33 33 10.000  4.799  1.446    3.533 1457.3  -2.9035  1455.4   -73.8  2.2563E-02   423.0     54.5
   THE BASE PRESSURE IS    2.790 (LBF/IN2) AND THE BASE THRUST IS    24.45 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5735.7 (LBF)

    THE NOZZLE WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =  28.551 DEGREES AND YE =  4.347 IN
23 52 10.000  4.347  3.396   10.090  8617.0   7.358  8546.1  1103.5  8.7112E-03  2780.0   19.204   5645.5   5646.1  293.743      -2
        0 CHARACTERISTICS CROSSED IN THE NOZZLE.
    THE BOATTAIL WALL CONTOUR IS A SECOND-ORDER POLYNOMIAL WITH THETAT =   3.661 DEGREES AND YE =  4.670 IN
33 33 10.000  4.670  1.456    3.484 1464.2  -3.6602  1461.2   -93.5  2.2341E-02   421.3     66.0
   THE BASE PRESSURE IS    2.806 (LBF/IN2) AND THE BASE THRUST IS    25.75 (LBF)
   THE TOTAL NOZZLE-BASE-BOATTAIL THRUST IS   5737.8 (LBF)

        AT THE  1 STEP   THETA = 28.551   YE =  4.347   THETA =  3.661   YE =  4.670   THRUST =    5737.76
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Figure F-5. Output for Sample Case 5 (Concluded).

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